# Inflation and the Very Early Universe

Georges Obied

University of Oxford

Morning of Theoretical Physics

25 Feb. 2023

Cosmic

# Inflation and the Very Early Universe

Georges Obied

University of Oxford

Morning of Theoretical Physics

25 Feb. 2023

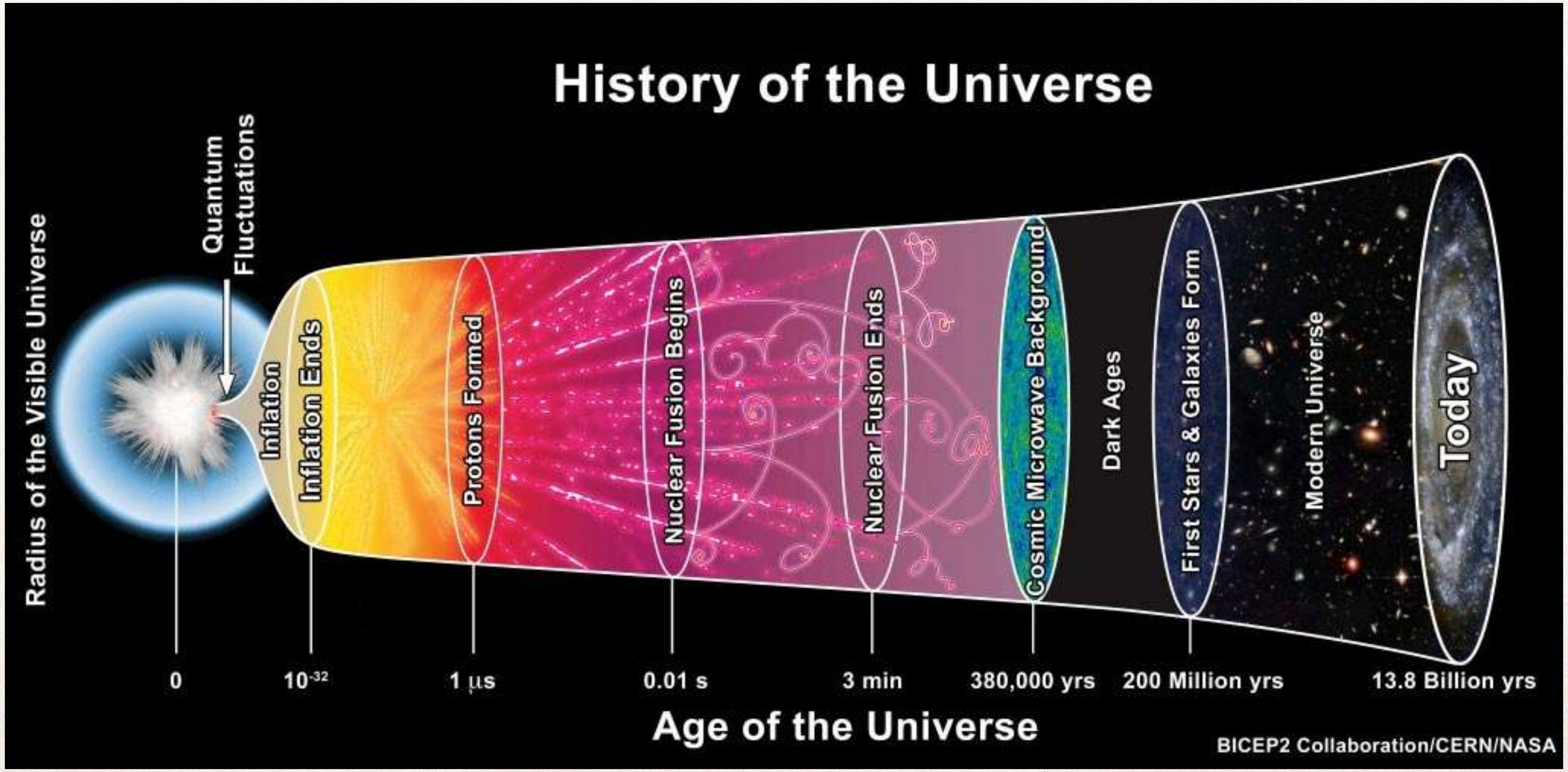
# Motivation and Overview

# Motivation and Overview

- **Inflation:** a period of accelerated expansion in the early universe.

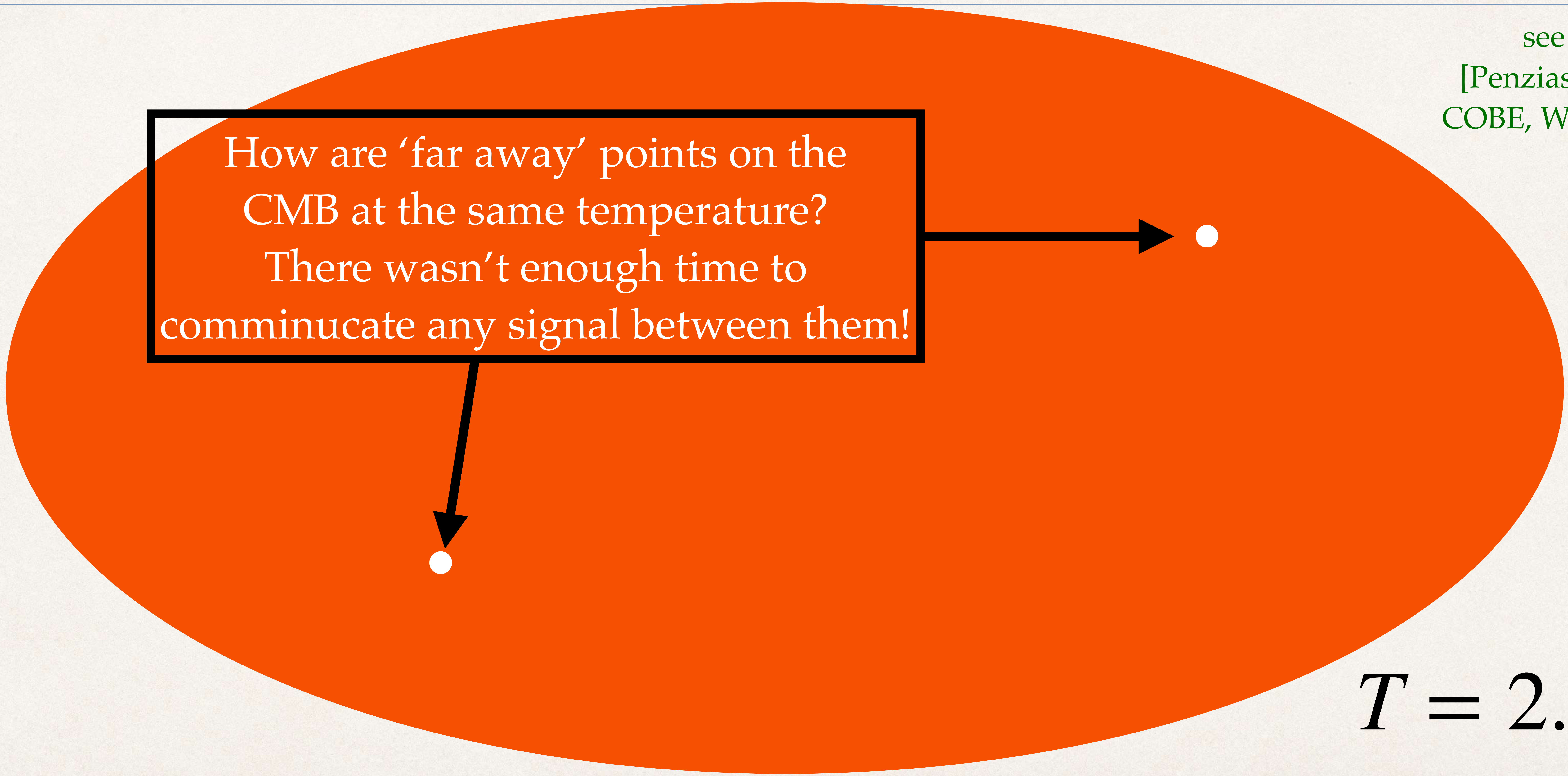
[Guth '87, Linde '87, Albrecht, Steinhardt '87]

- It explains some puzzling observations about our universe



# The Cosmic Microwave Background

see for e.g.:
[Penzias, Wilson ’65,
COBE, WMAP, Planck]




$T = 2.7\ \mathrm{K}$

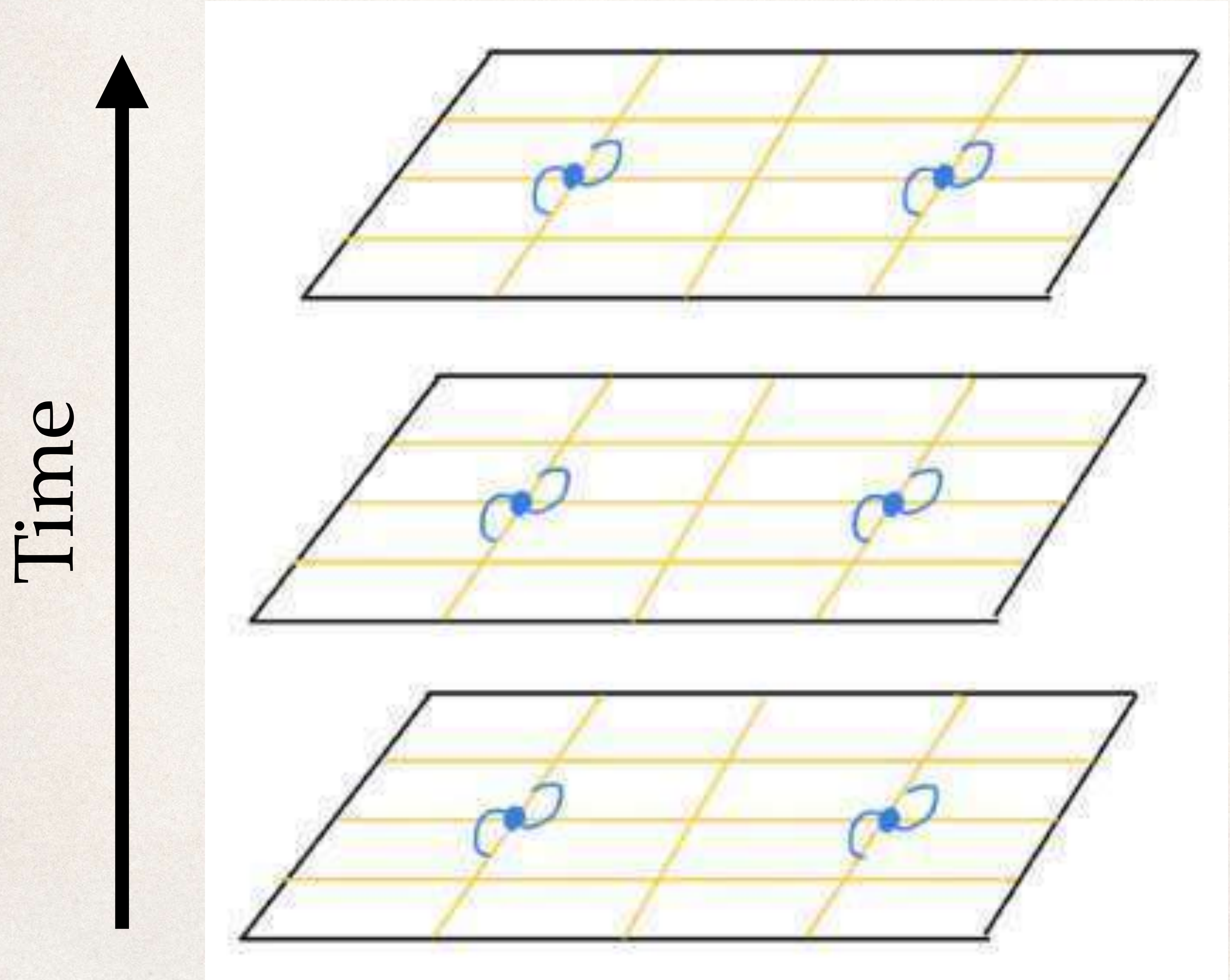
The question we want to answer is: How is the CMB so uniform?

# Outline

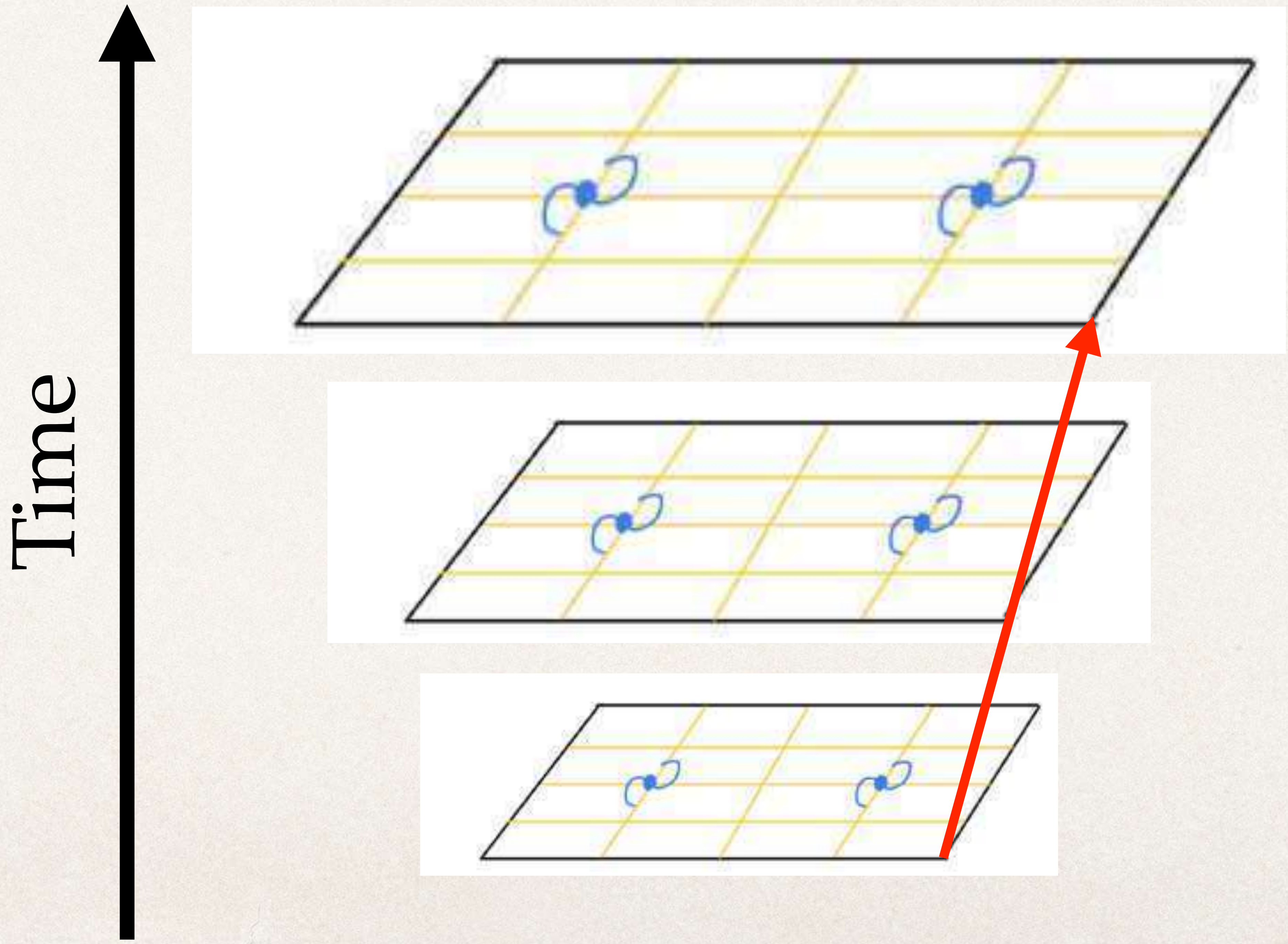
- The expanding universe
- The particle horizon
- The horizon problem
- The inflationary paradigm
- Quantum Perturbations (very briefly)
- Conclusion

# The expanding universe

- In Special Relativity, the ‘interval’ between two points is:
- $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$




- In cosmology, we describe an expanding universe using:
- $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$



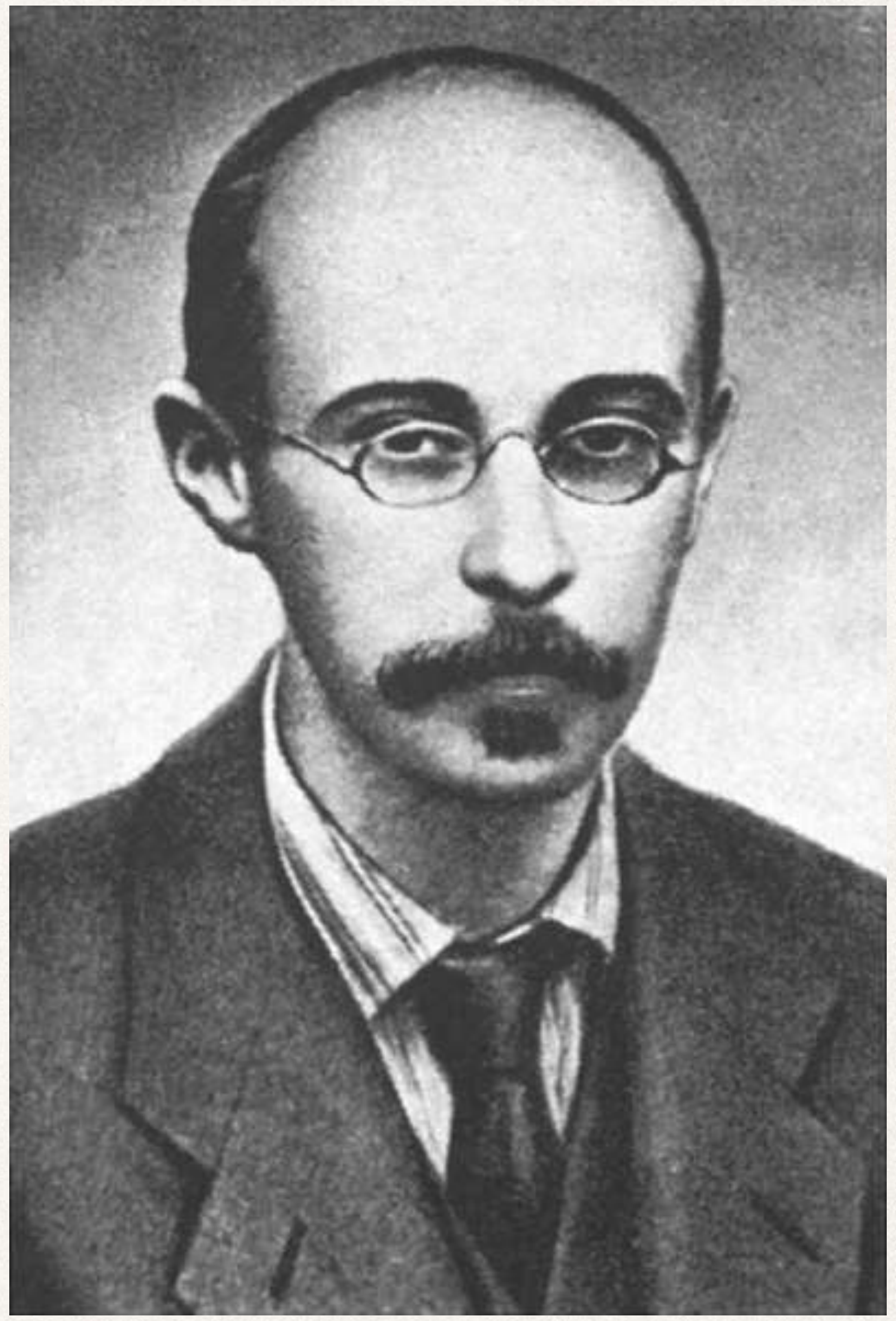
# The expanding universe

- The evolution of $a(t)$ is dictated by the contents of the universe.

- From Einstein's general relativity, we get the differential equation (called the first Friedmann equation) :

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(a)$$

where $\rho(a)$ is the energy density and $H$ is called the Hubble rate.



Alexander Friedmann

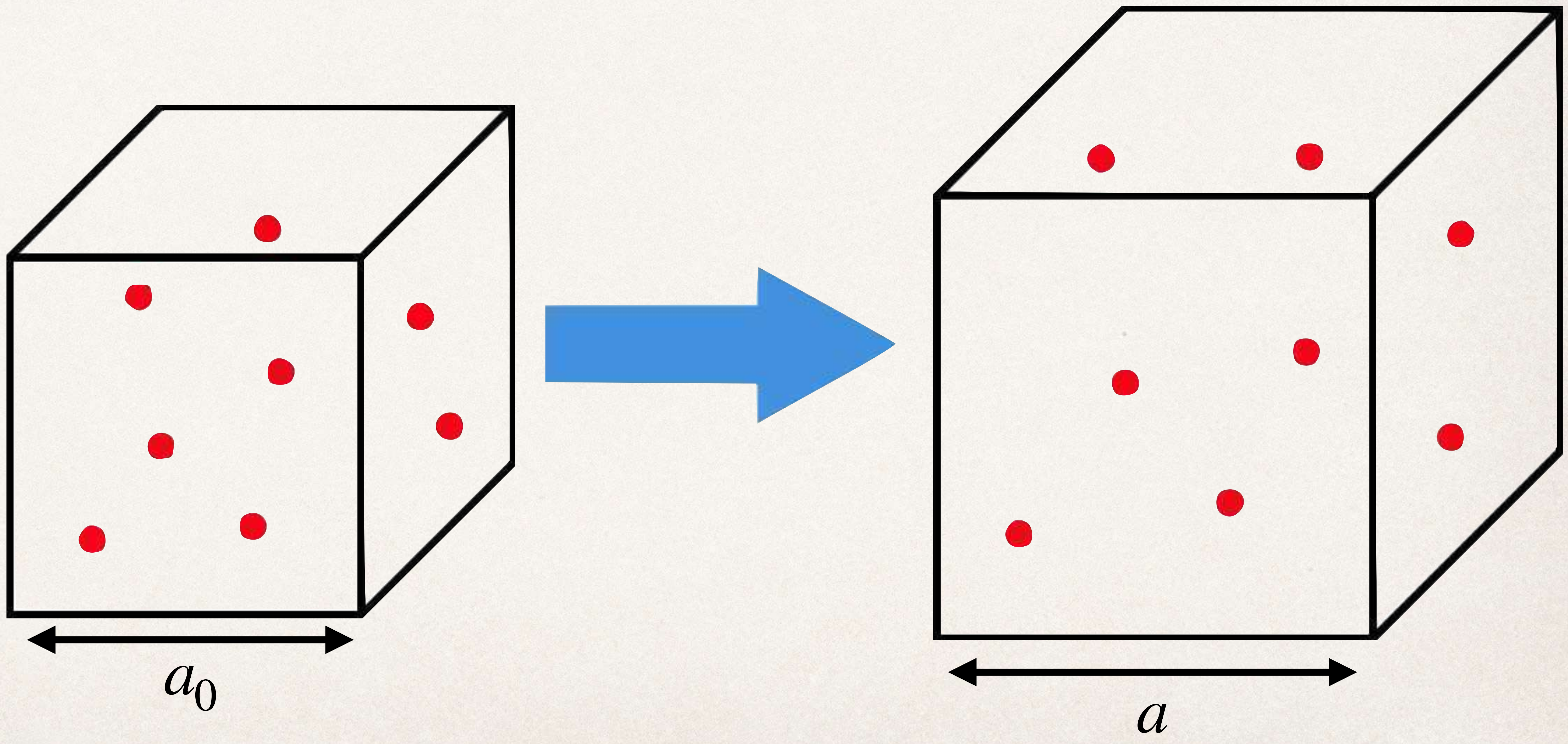
# The expanding universe: matter

- Consider a box with side length equal to one coordinate unit filled with matter (i.e. massive particles at rest)

Let the density today be $\rho_0$

Then the density at a different time with scale factor $a$ is:

$$\rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^3$$



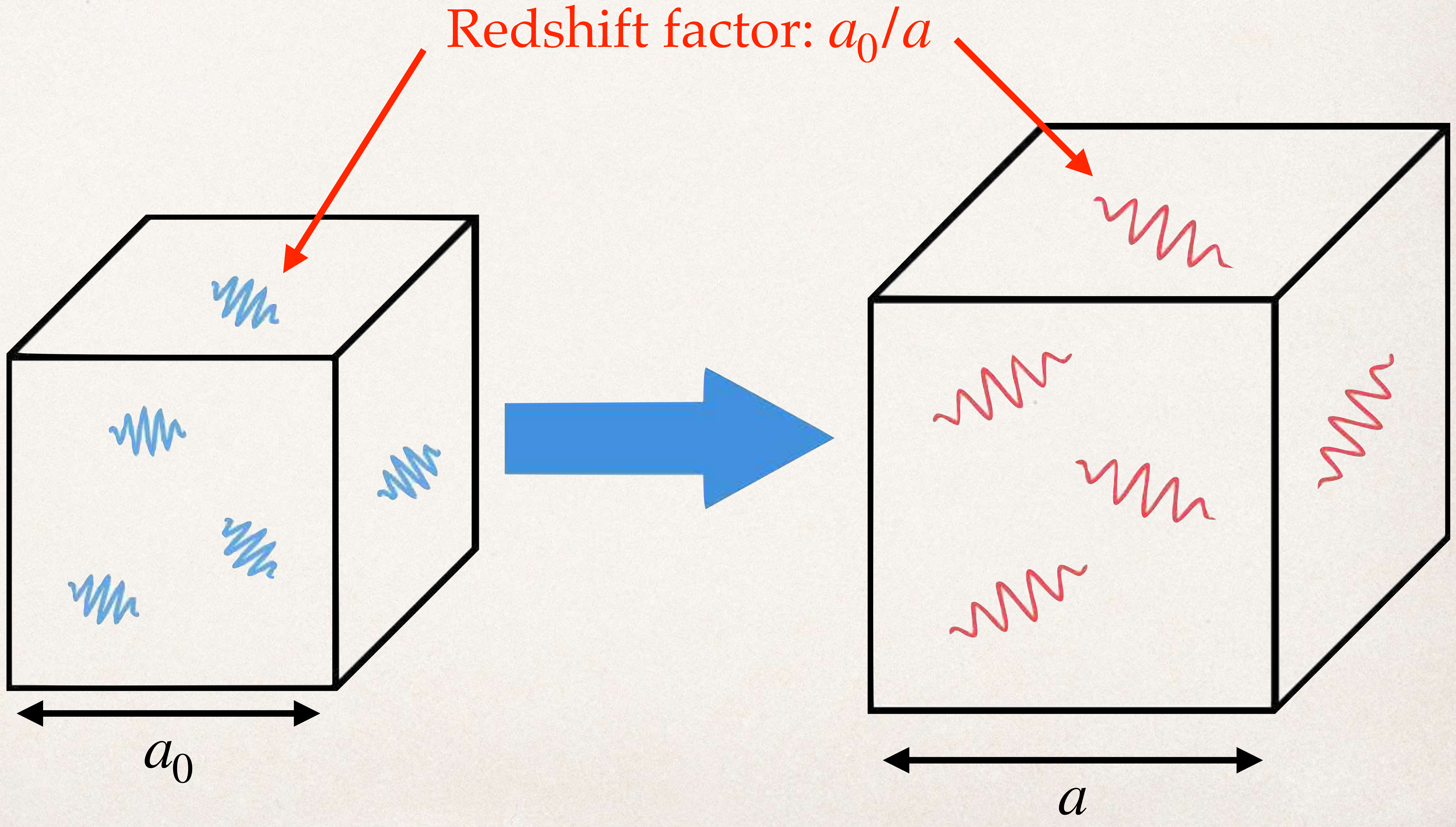
# The expanding universe: radiation

- We have a similar story for a box filled with radiation (i.e. highly relativistic particles like photons)

Let the density today be $\rho_0$

Then the density at a different time with scale factor $a$ is:

$$\rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^3 \left(\frac{a_0}{a}\right)$$
$$= \rho_0 \left(\frac{a_0}{a}\right)^4$$



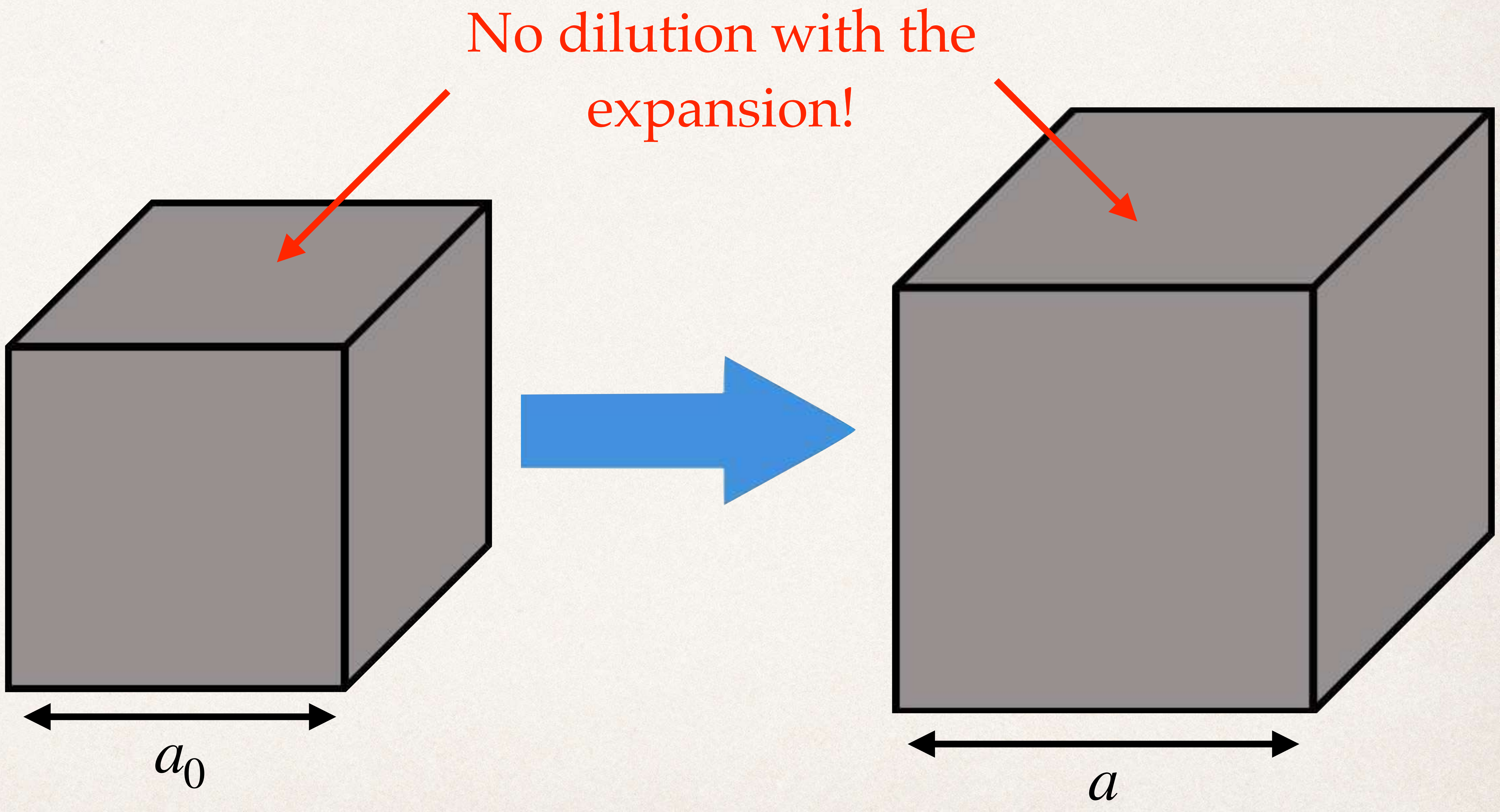
# The expanding universe: Dark Energy

- For a box 'filled' with Dark Energy

Let the density today be $\rho_0$

Then the density at a different time with scale factor $a$ is:

$$\rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^0 = \rho_0$$



# Our expanding universe

- In summary, we can write

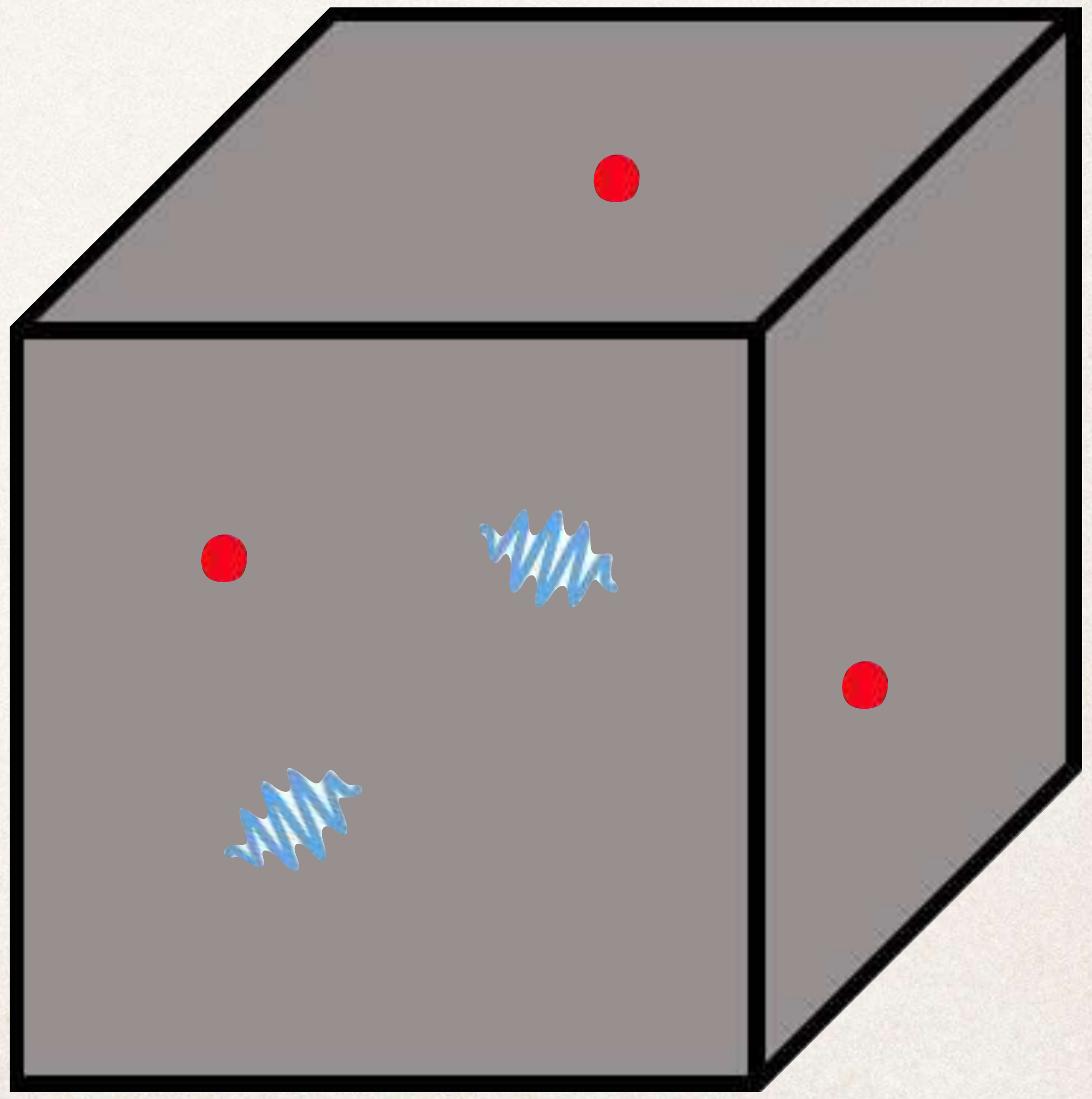
$$\rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+w)}$$

where $w$ takes on different values for matter, radiation and dark energy:

$$w_{\rm m} = 0 \qquad ; \qquad w_{\rm r} = \frac{1}{3} \qquad ; \qquad w_{\rm de} = -1$$

- In our Universe, we have multiple components:

$$\rho_{\rm tot}(a) = \rho_{\rm m,0} \left(\frac{a_0}{a}\right)^3 + \rho_{\rm r,0} \left(\frac{a_0}{a}\right)^4 + \rho_{\rm de,0}$$



# Toy expanding universes

- Given $\rho(a)$, we can now get some intuition by solving the Friedmann equation with one component in the energy density:

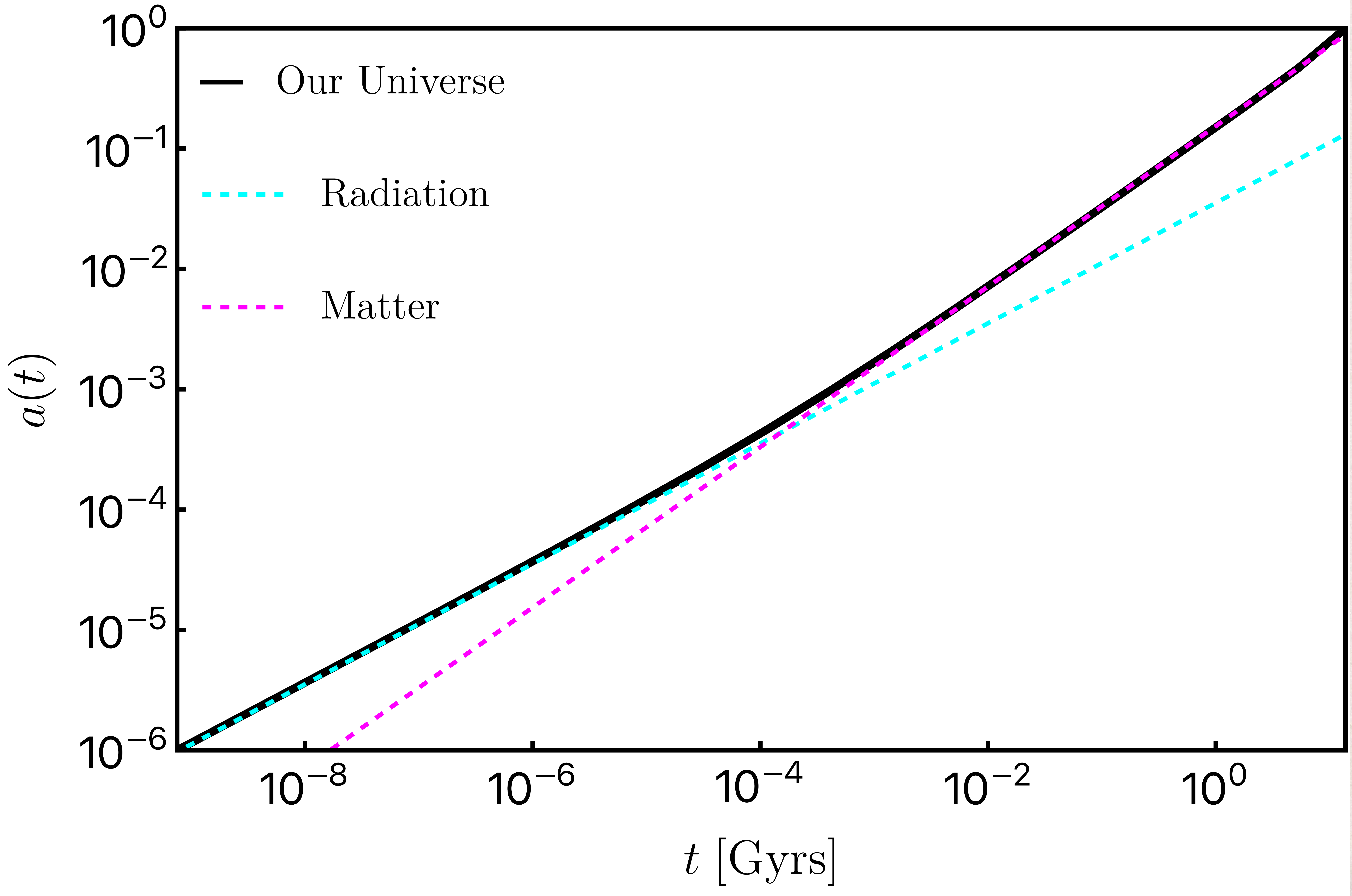
$$\frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi G}{3}\rho_0}\left(\frac{a_0}{a(t)}\right)^{\frac{3}{2}(1+w)} \implies \int \left(\frac{a}{a_0}\right)^{\frac{1}{2}(1+3w)} d\left(\frac{a}{a_0}\right) = \sqrt{\frac{8\pi G}{3}\rho_0}\int dt$$

$$\implies a(t) \propto \begin{cases} t^{2/3} & \text{for matter} \\ t^{1/2} & \text{for radiation} \\ e^{Ht} & \text{for Dark Energy} \end{cases}$$

# Our expanding universe

- In our universe, we have a combination of matter, radiation, and Dark Energy:

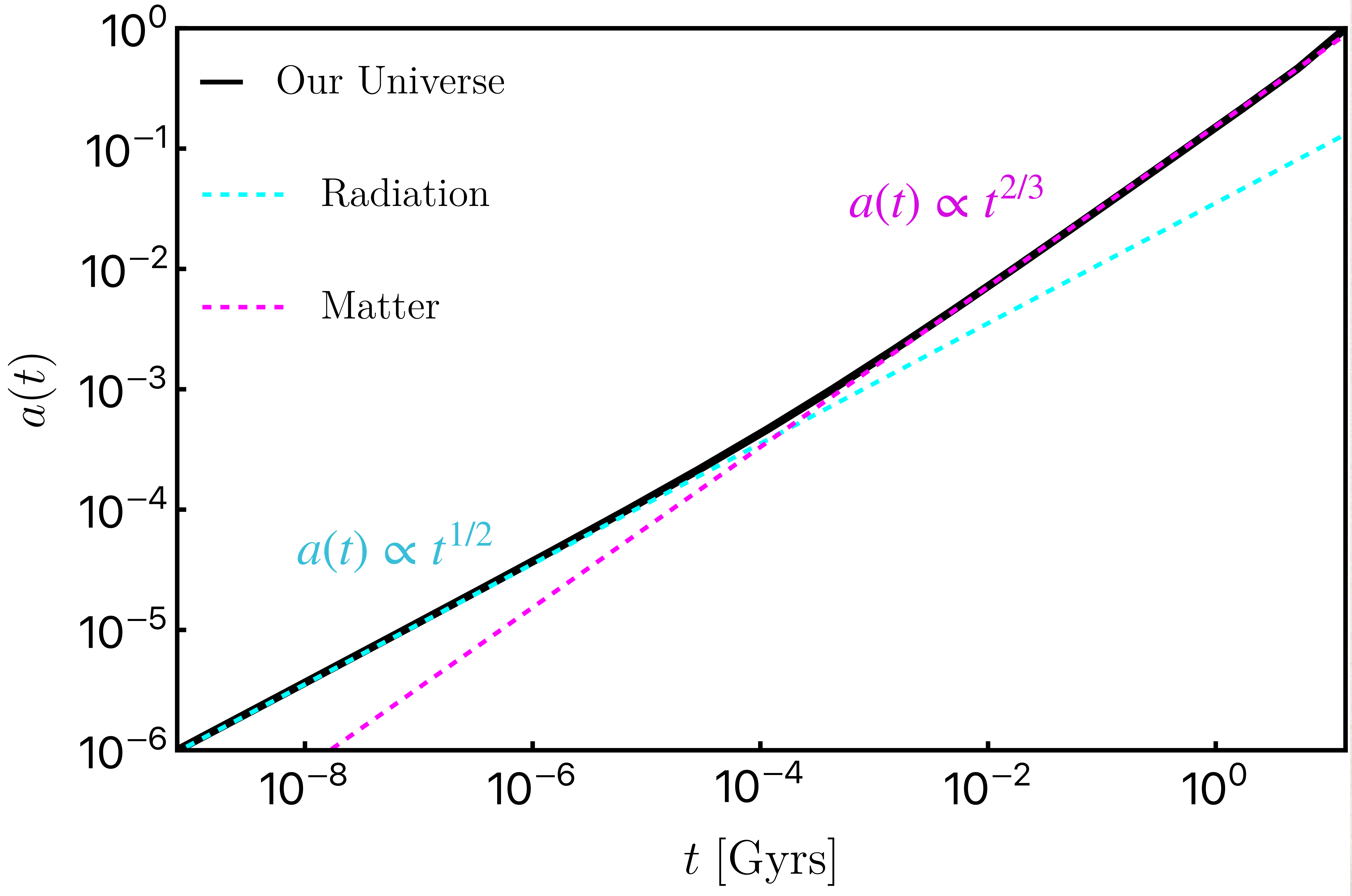
$$\rho_{\text{tot}} = \rho_{\text{m},0}\left(\frac{a_0}{a}\right)^3 + \rho_{\text{r},0}\left(\frac{a_0}{a}\right)^4 + \rho_{\text{de},0}$$



# Our expanding universe

- In our universe, we have a combination of matter, radiation, and Dark Energy:

$$\rho_{\rm tot} = \rho_{\rm m,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\rm r,0}\left(\frac{a_0}{a}\right)^4 + \rho_{\rm de,0}$$

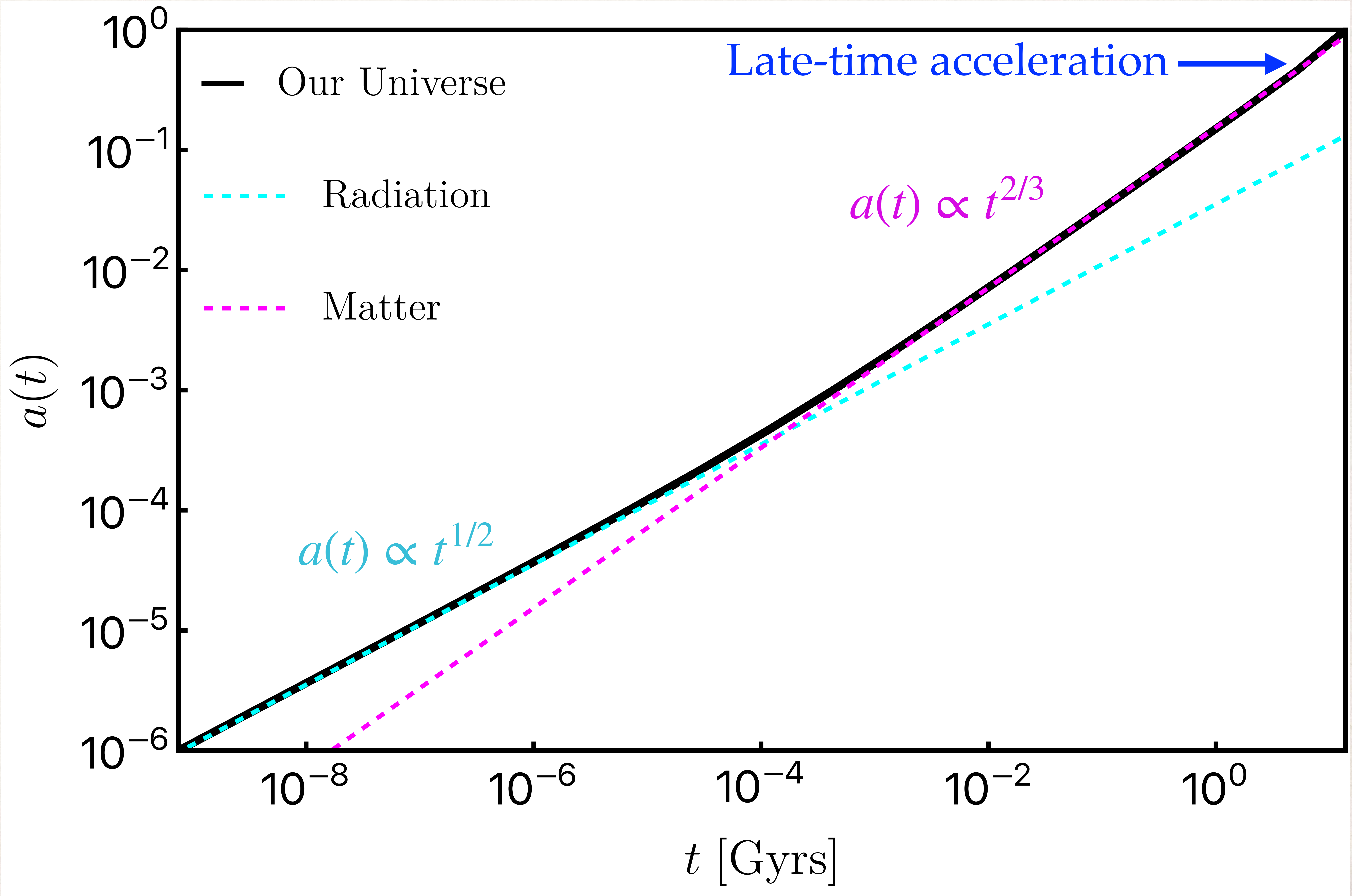


# The expanding universe

- In our universe, we have a combination of matter, radiation, and Dark Energy:

$$\rho_{\rm tot} = \rho_{\rm m,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\rm r,0}\left(\frac{a_0}{a}\right)^4 + \rho_{\rm de,0}$$

This is responsible for late-time cosmic acceleration but plays no role in the early universe



# The particle horizon

- Largest distance that light could have travelled in the age of the universe:

$$D(t_f) = a(t_f) \int_0^{t_f} \frac{c\,dt}{a(t)} = ca(t_f) \int_0^{a(t_f)} \frac{da}{a\dot{a}} \implies D(a) = ca \int_0^{a} \frac{d\tilde{a}}{\tilde{a}} \frac{1}{\dot{\tilde{a}}}$$

Multiply by scale factor to get physical Distance

Coordinate distance

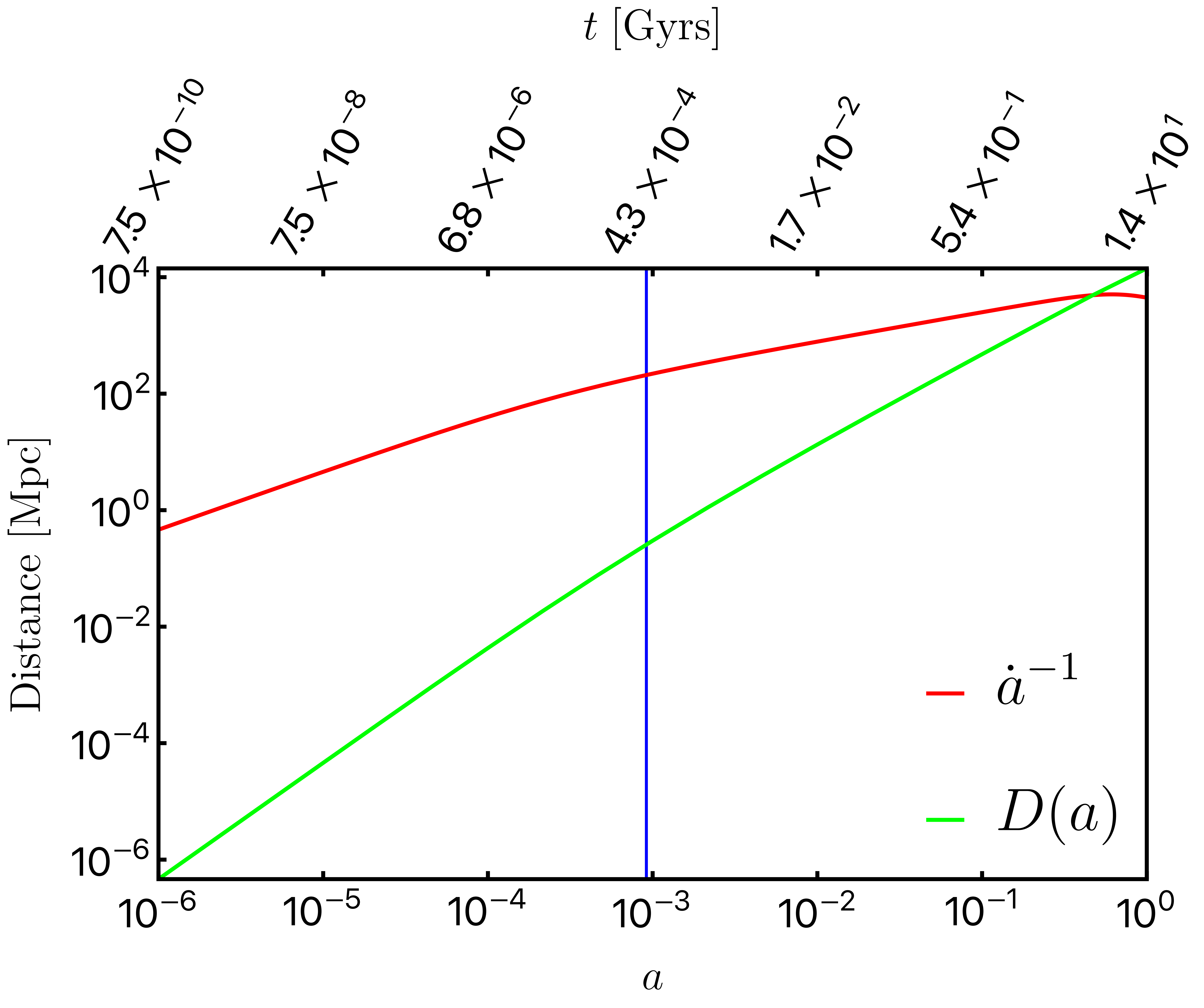
- Notice that $1/\dot{a}$ is always increasing for 'normal' substances ($w_{\mathrm{m}} = 0$ or $w_{\mathrm{r}} = 1/3$).

- This means that $D(a)$ is dominated by $1/\dot{a}$ in the late universe.

# The horizon

$$D(a) = a \int_0^a \frac{d\tilde{a}}{\tilde{a}} \frac{1}{\dot{\tilde{a}}}$$

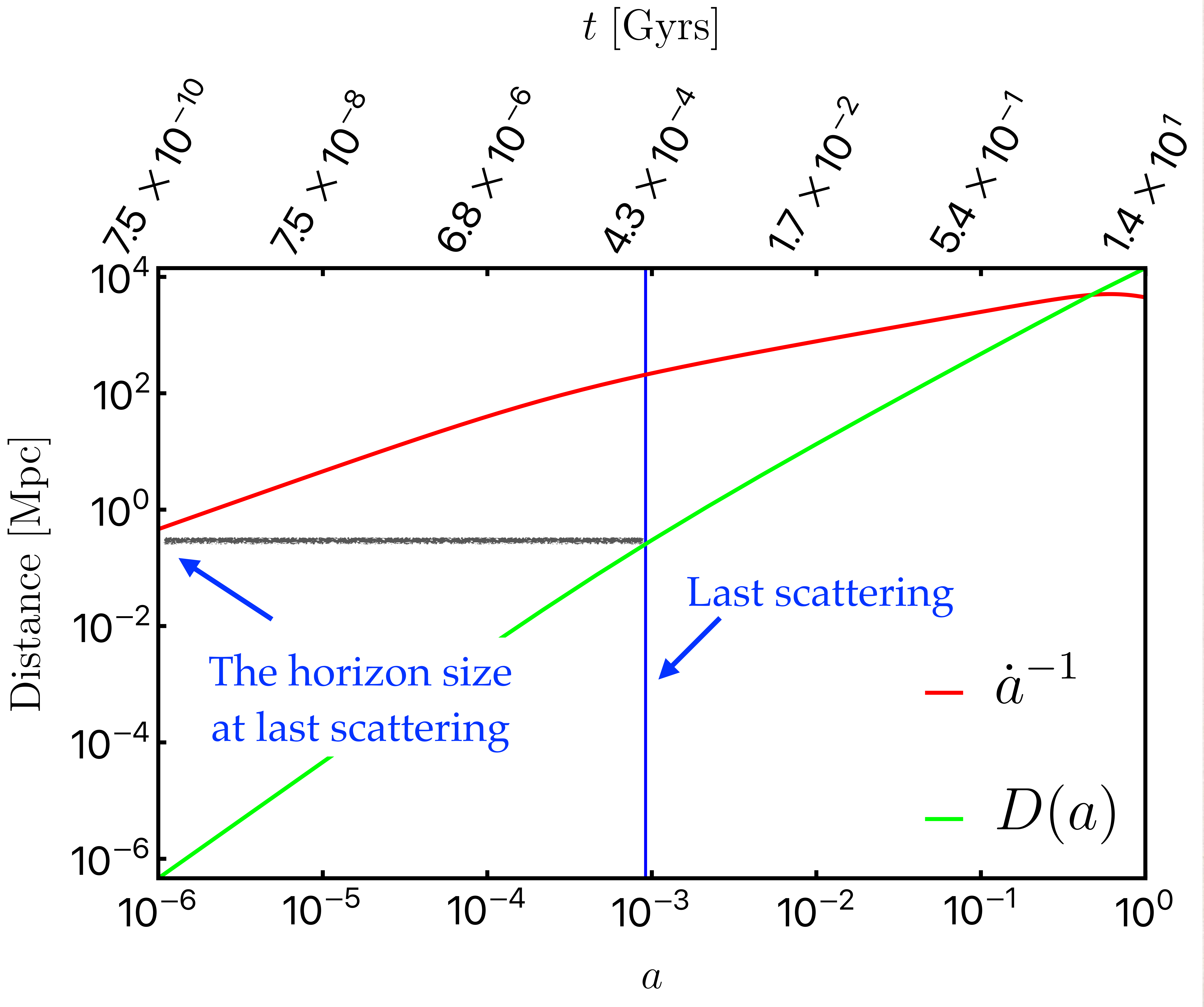
- Since $\dot{a}^{-1}$ is always increasing, the most important contributions to $D(a)$ come from late times:



# The horizon

$$D(a) = a\int_0^a \frac{d\tilde{a}}{\tilde{a}}\frac{1}{\dot{\tilde{a}}}$$

- Since $\dot{a}^{-1}$ is always increasing, the most important contributions to $D(a)$ come from late times:



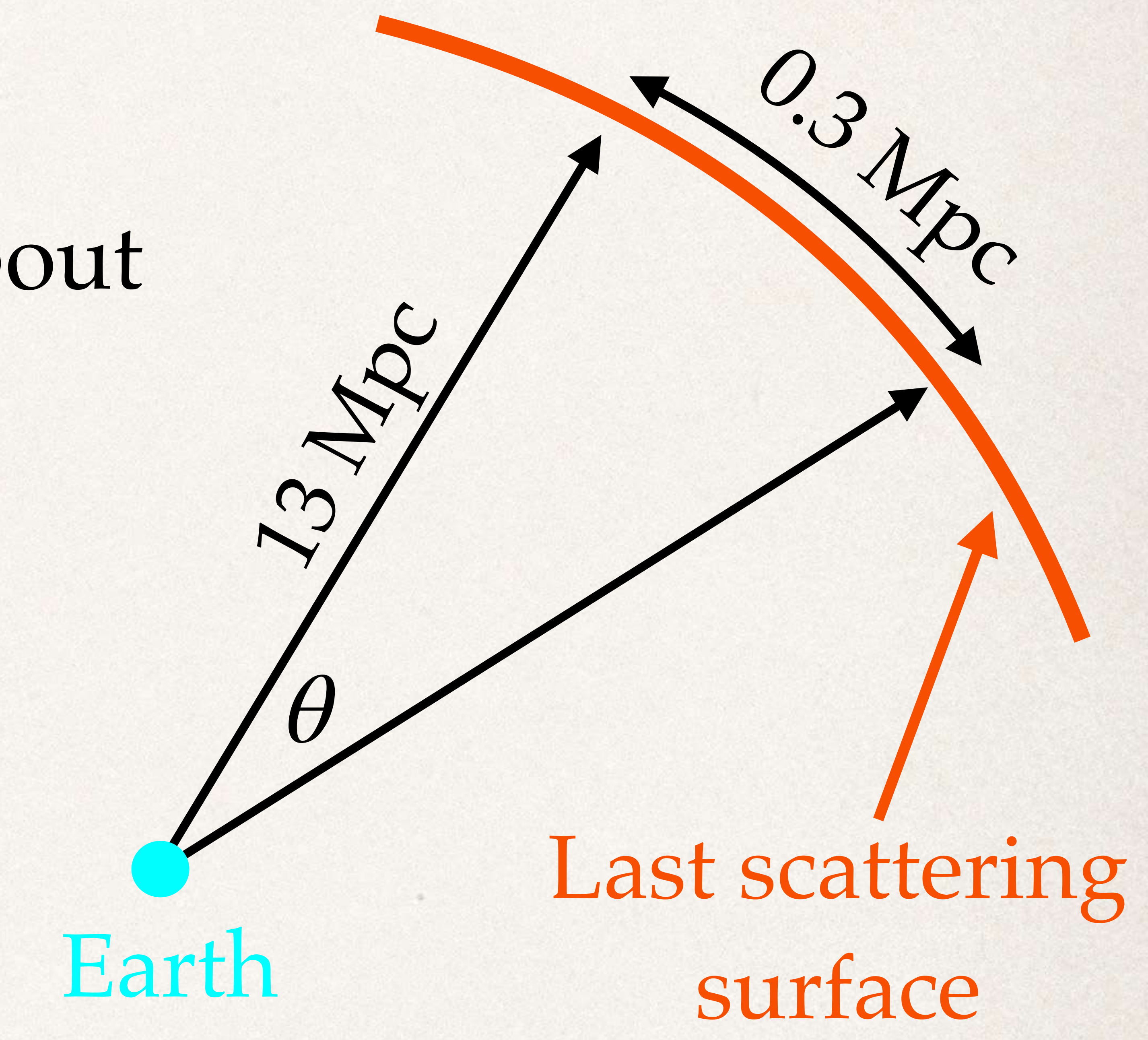
# The horizon problem

- At the time of last scattering the horizon size was about 0.3 Mpc
- Our (angular diameter) distance to the last scattering surface is about 13 Mpc
- The angle subtended by a causally connected patch is then:

$$\theta \approx \frac{0.3}{13}\ \mathrm{rad} \approx 0.023\ \mathrm{rad}$$

- The number of causally disconnected patches on the CMB sky is then

$$N \approx \frac{4\pi}{0.023^2} \sim 24000$$



# Football match analogy

- The Den Stadium which has about 20000 seats.


Image Credit:
Groundhopper Soccer Guides

# Football match analogy

- The Den Stadium which has about 20000 seats.


Francesco


Andy


Image Credit:
Groundhopper Soccer Guides

# Football match analogy

- The Den Stadium which has about 20000 seats.

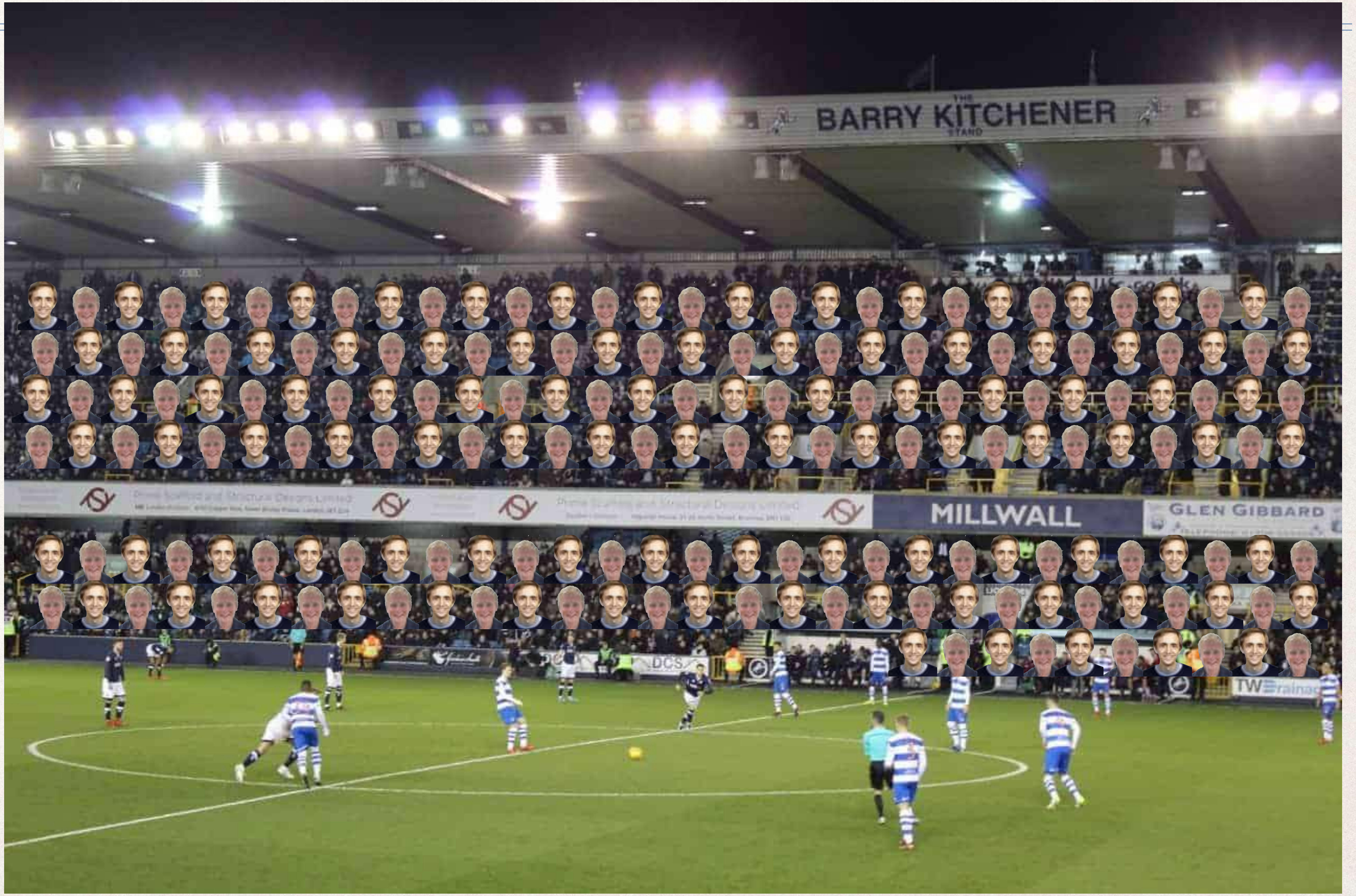
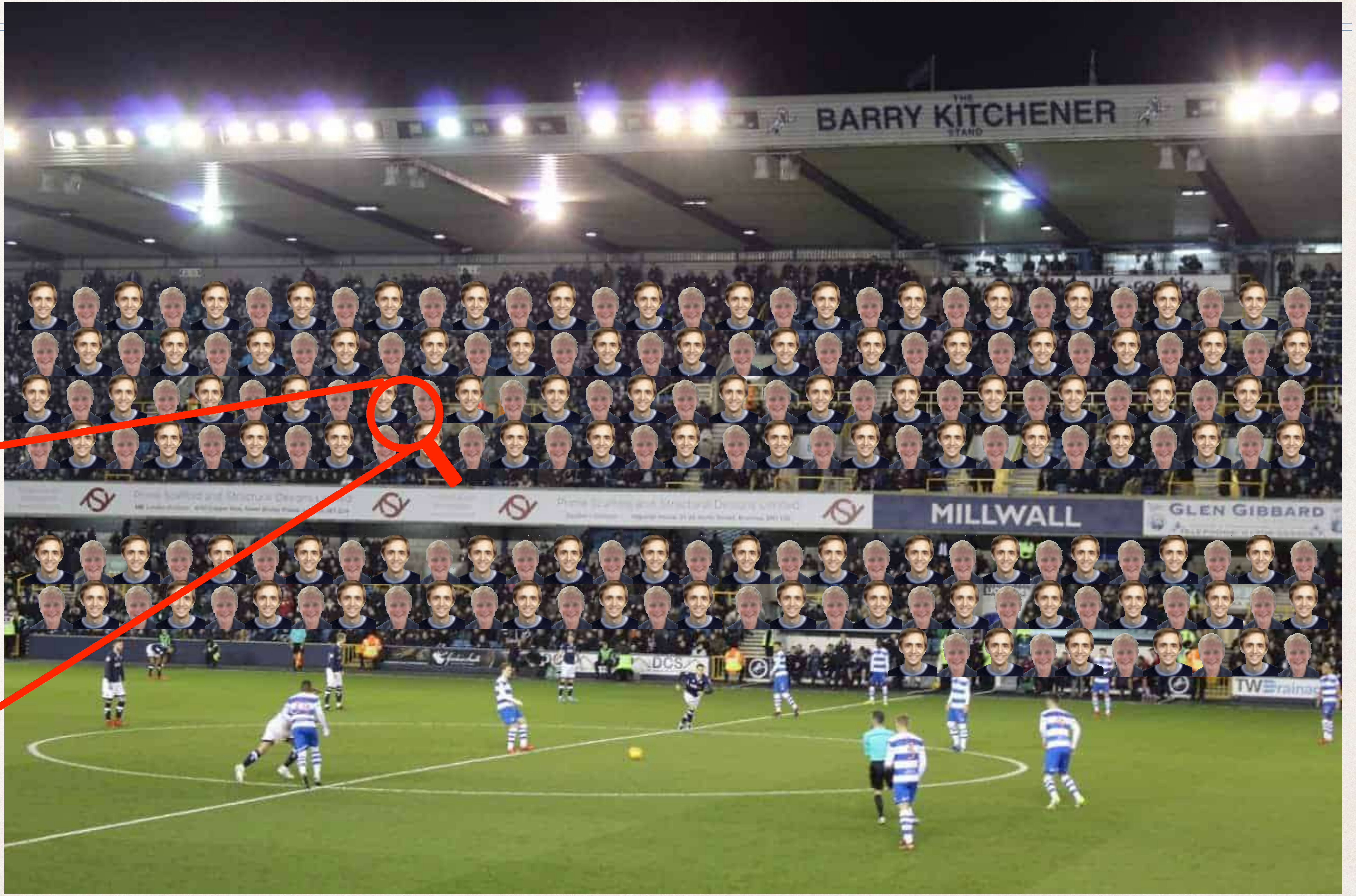



Image Credit:
Groundhopper Soccer Guides

# Football match analogy

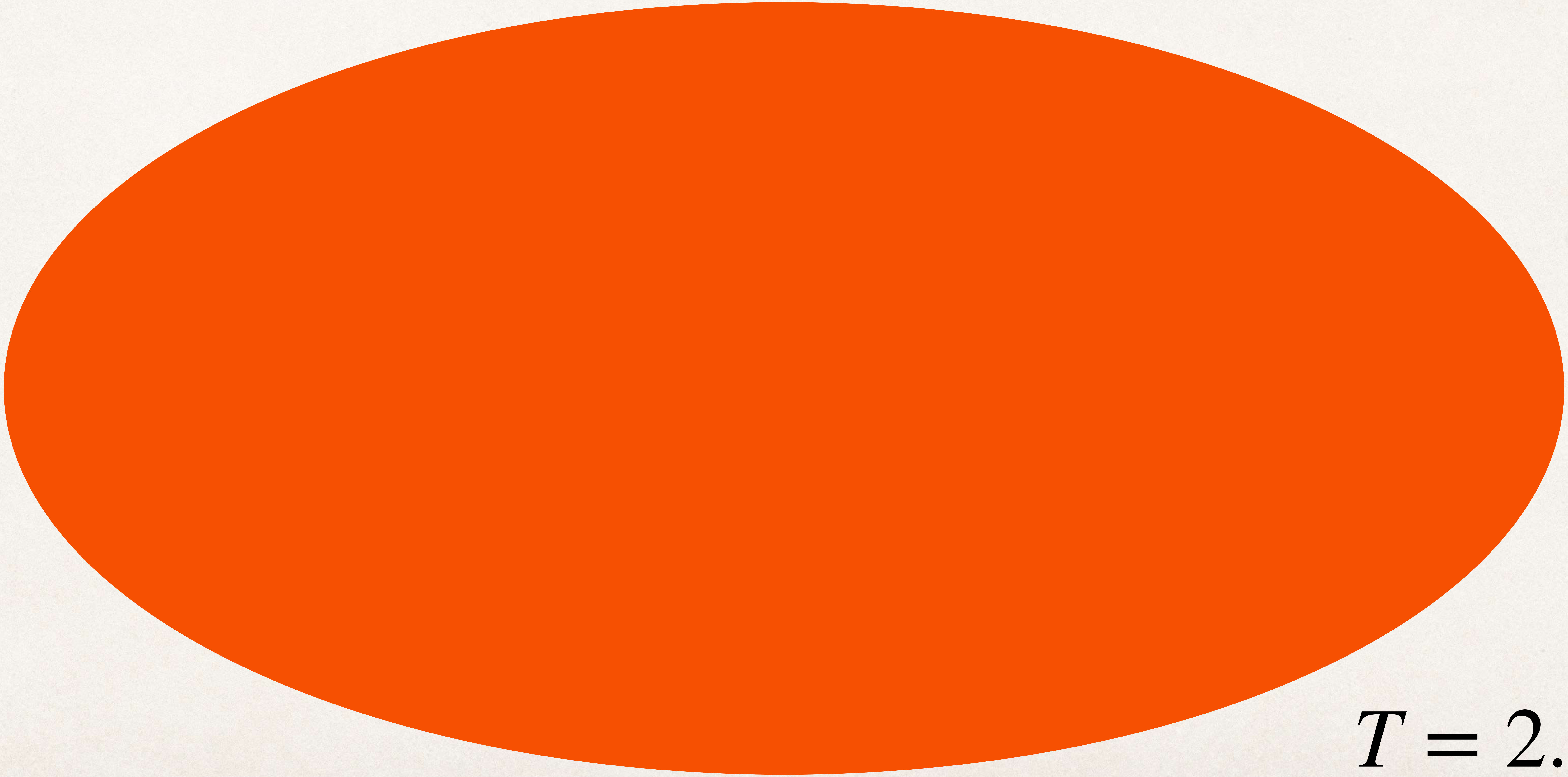
- The Den Stadium which has about 20000 seats.




Come on guys!

Image Credit:
Groundhopper Soccer Guides

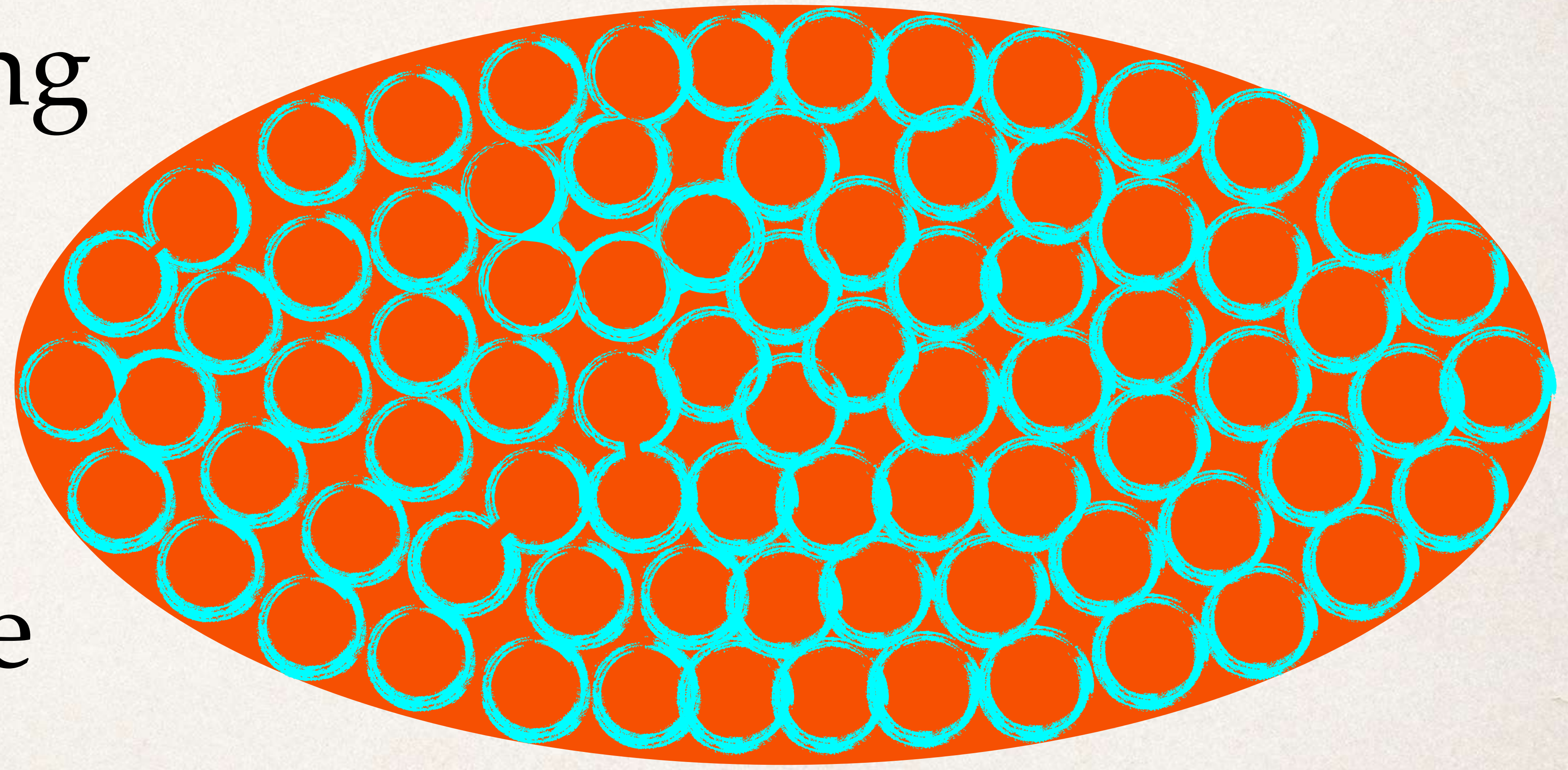
# Back to the CMB



$T = 2.7\ \mathrm{K}$

# Back to the CMB

- According to the Hot Big Bang model, there are 20k causally disconnected patches (i.e. patches that haven't communicated at any time in the past).
- Yet they all chose to go to the same temperature $T = 2.7K$
- One should be suspicious that the Hot Big Bang model vastly underestimates the true size of the horizon

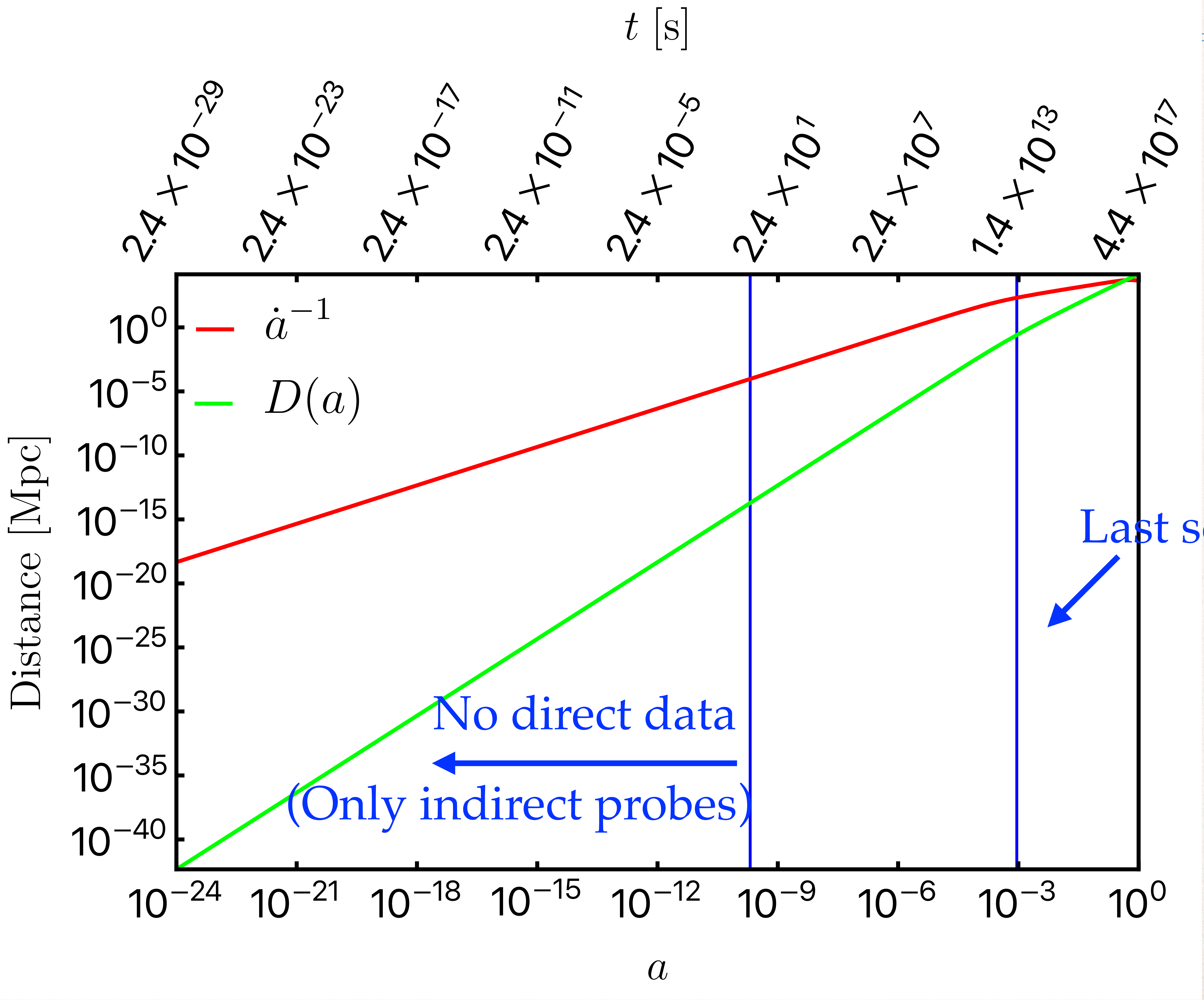


- In other words, we need to find a way to make sure that there are no causally disconnected patches in the CMB. This is what inflation does!

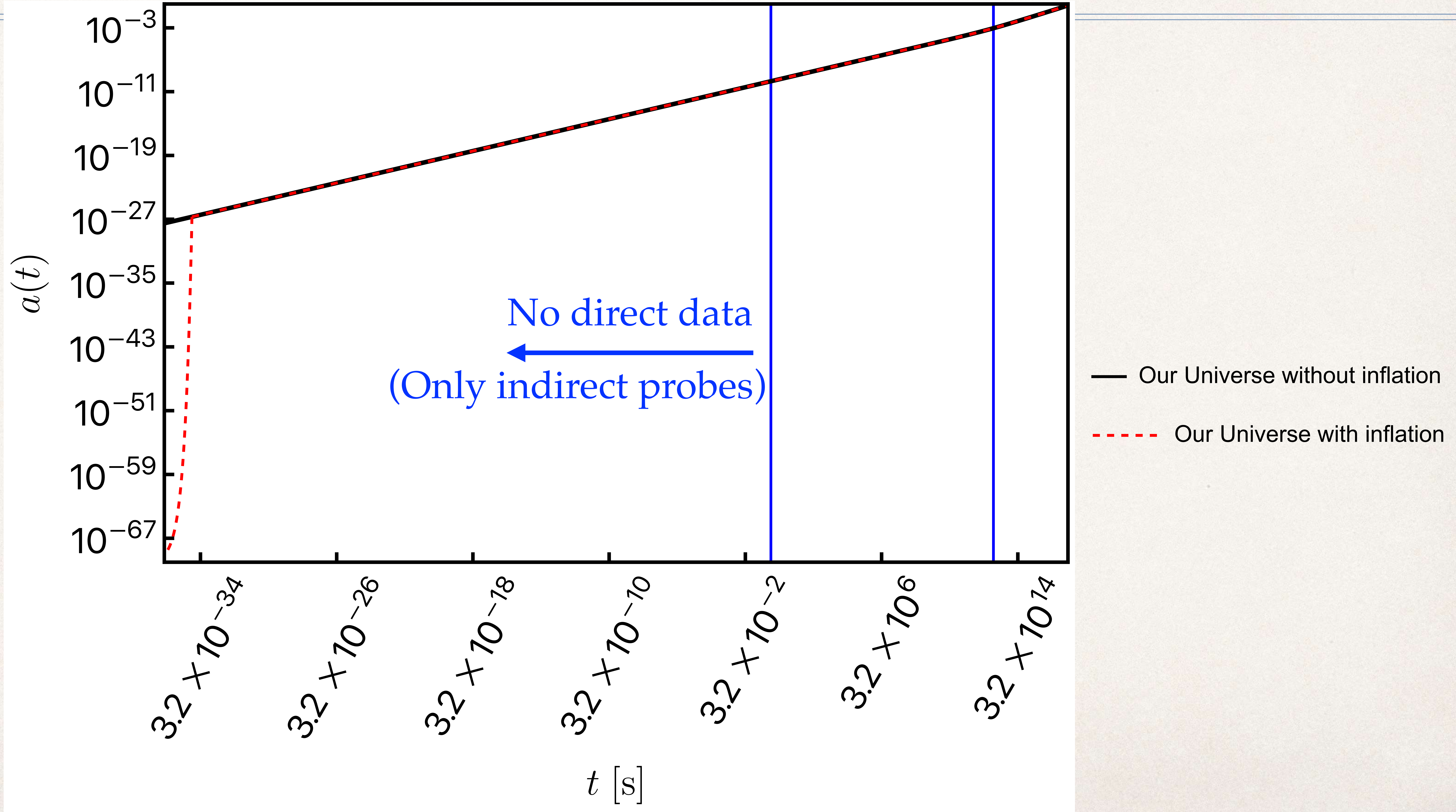
# Inflationary solution

- The problem is that the horizon receives largest contributions from late times.
- This means that the horizon is mostly what we would estimate from late time cosmology that is very well constrained by data.
- The horizon however can be much bigger if it received large contributions from early times
- This happens if $1/\dot{a}$ decreased in the past
- This happens in an accelerating $\ddot{a} > 0$ universe!

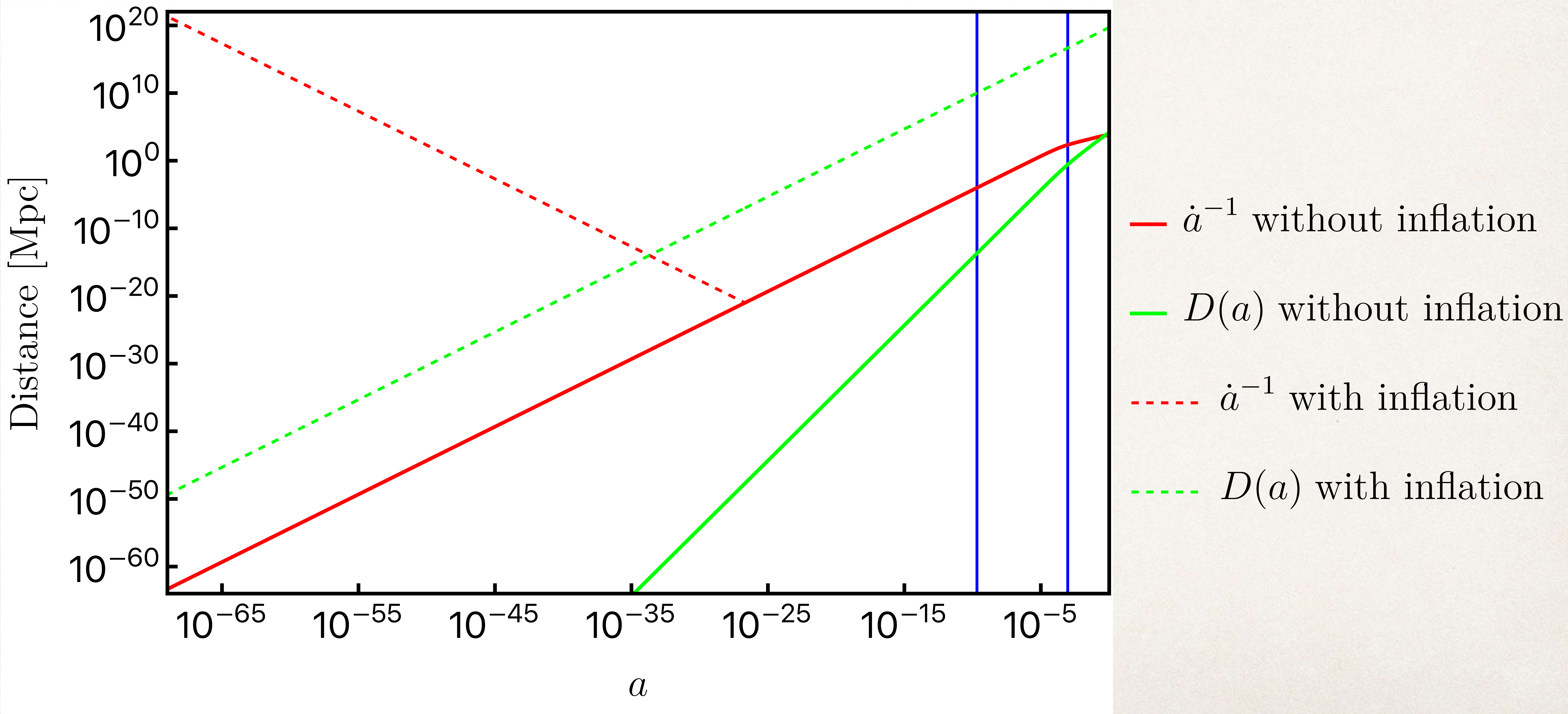
# Inflationary solution



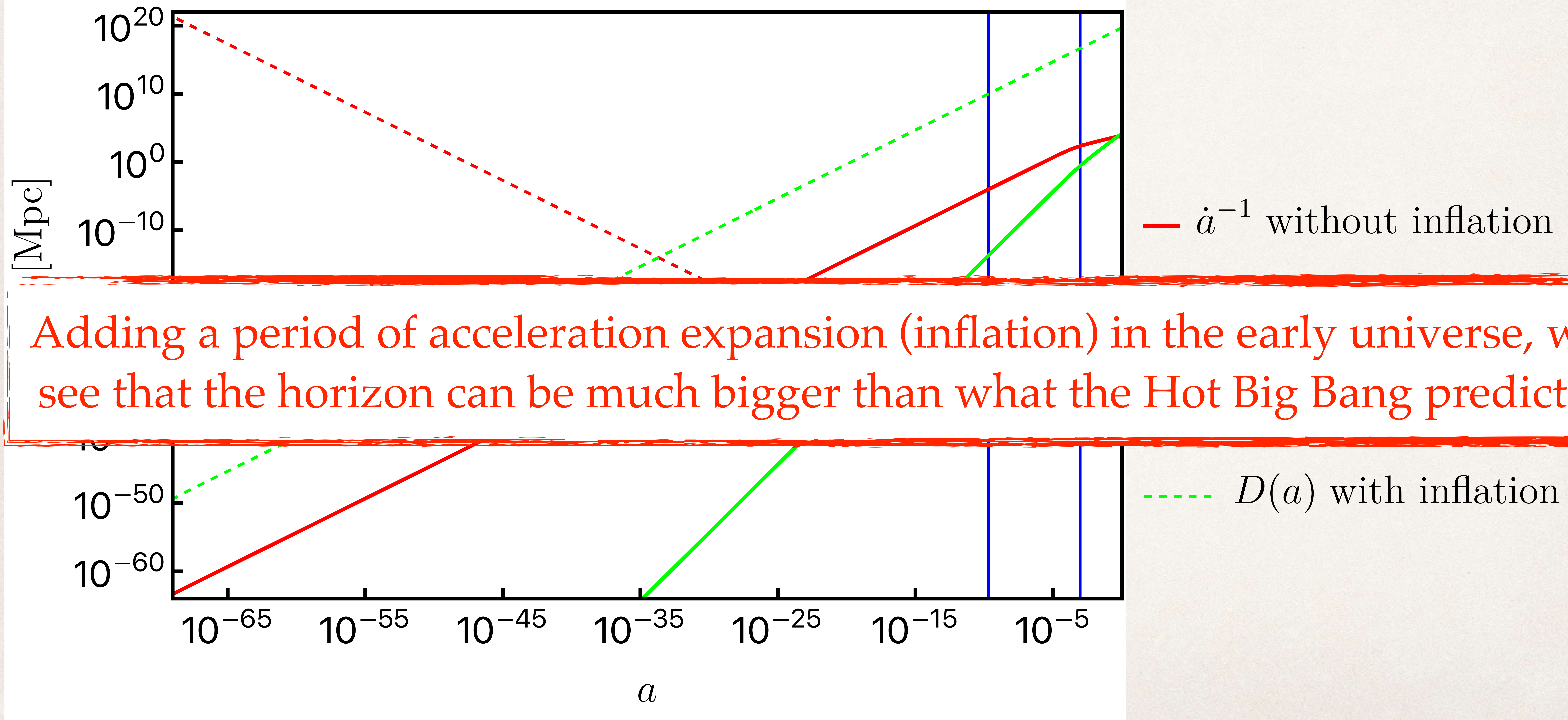
# Inflationary solution



# What does this do to the horizon?



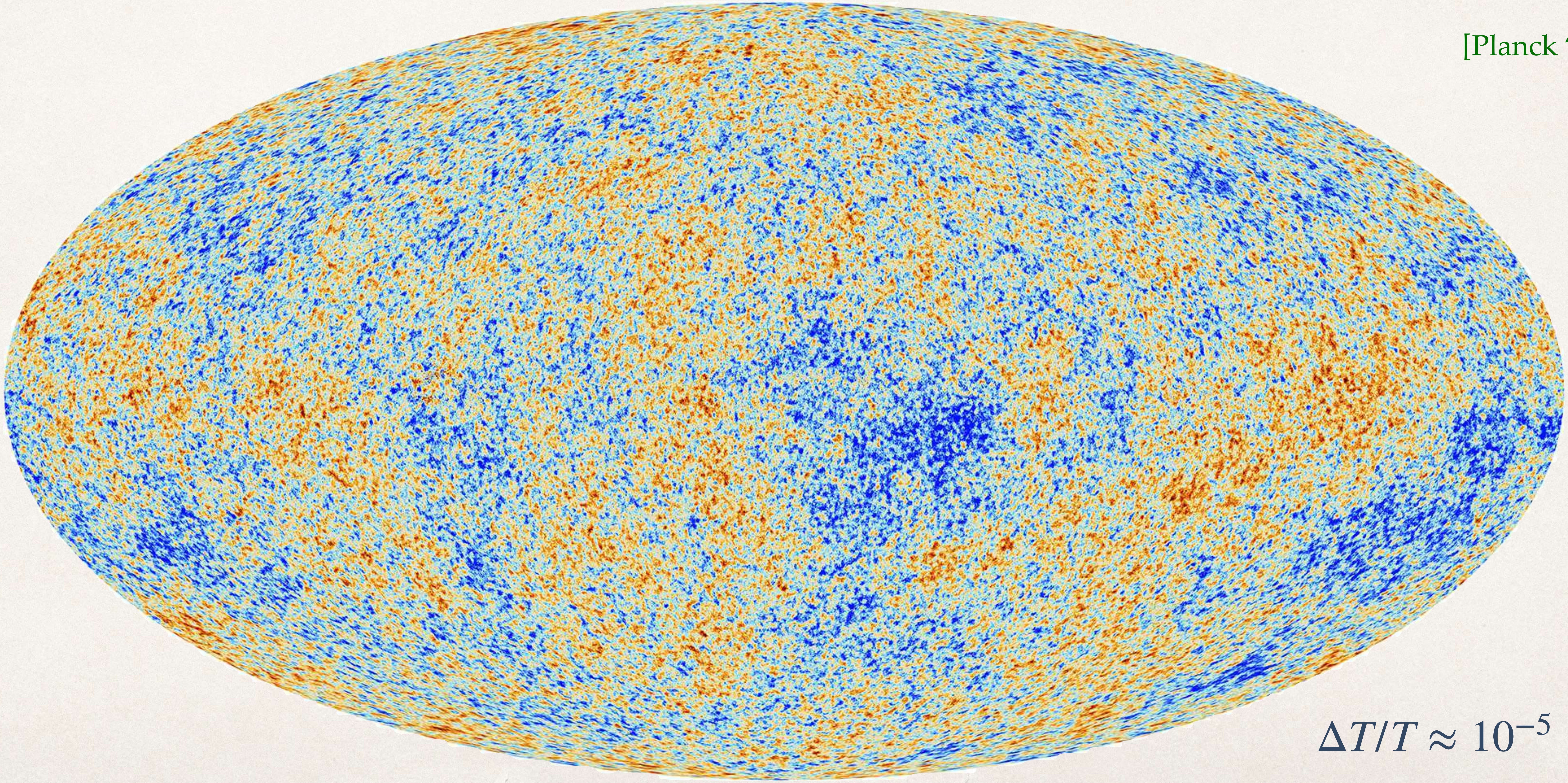
# What does this do to the horizon?




Adding a period of acceleration expansion (inflation) in the early universe, we see that the horizon can be much bigger than what the Hot Big Bang predicts.

# Perturbations

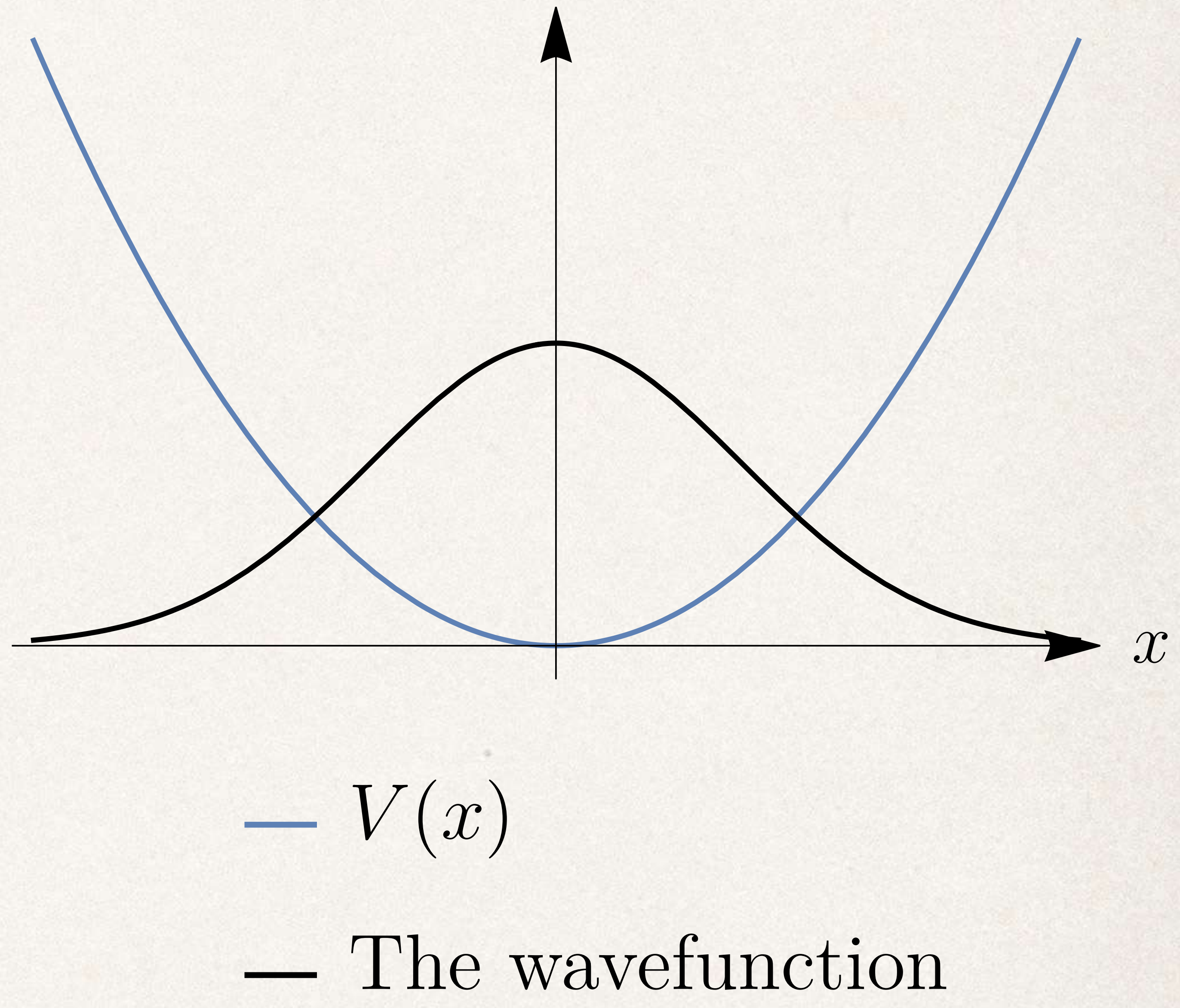
[Planck '18]



$\Delta T/T \approx 10^{-5}$

# Perturbations

- Recall that the ground state of the quantum simple harmonic oscillator (SHO) has a Gaussian wavefunction




- Even in the ground state, the SHO is not localized at one point and has fluctuations:

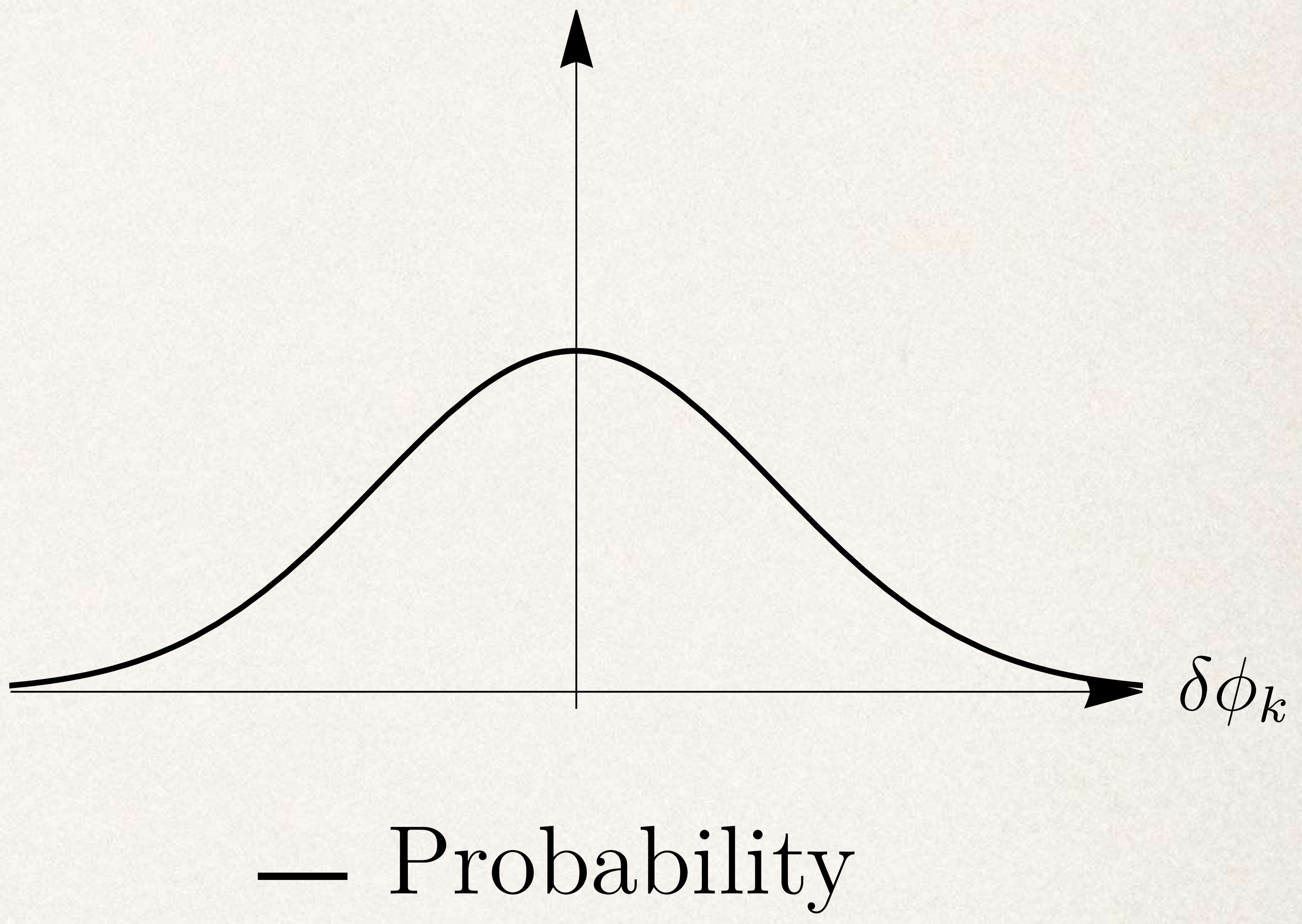
$$\langle 0 | \hat{x}^2 | 0 \rangle \neq 0$$

- So if we measure the position of the oscillating particle, we would typically get non-zero values.

# Perturbations

- In the simplest models of inflation, the energy density to drive the expansion is due to a scalar field:

$$\phi(t, \vec{x}) = \bar{\phi}(t) + \delta\phi(t, \vec{x})/a(t)$$

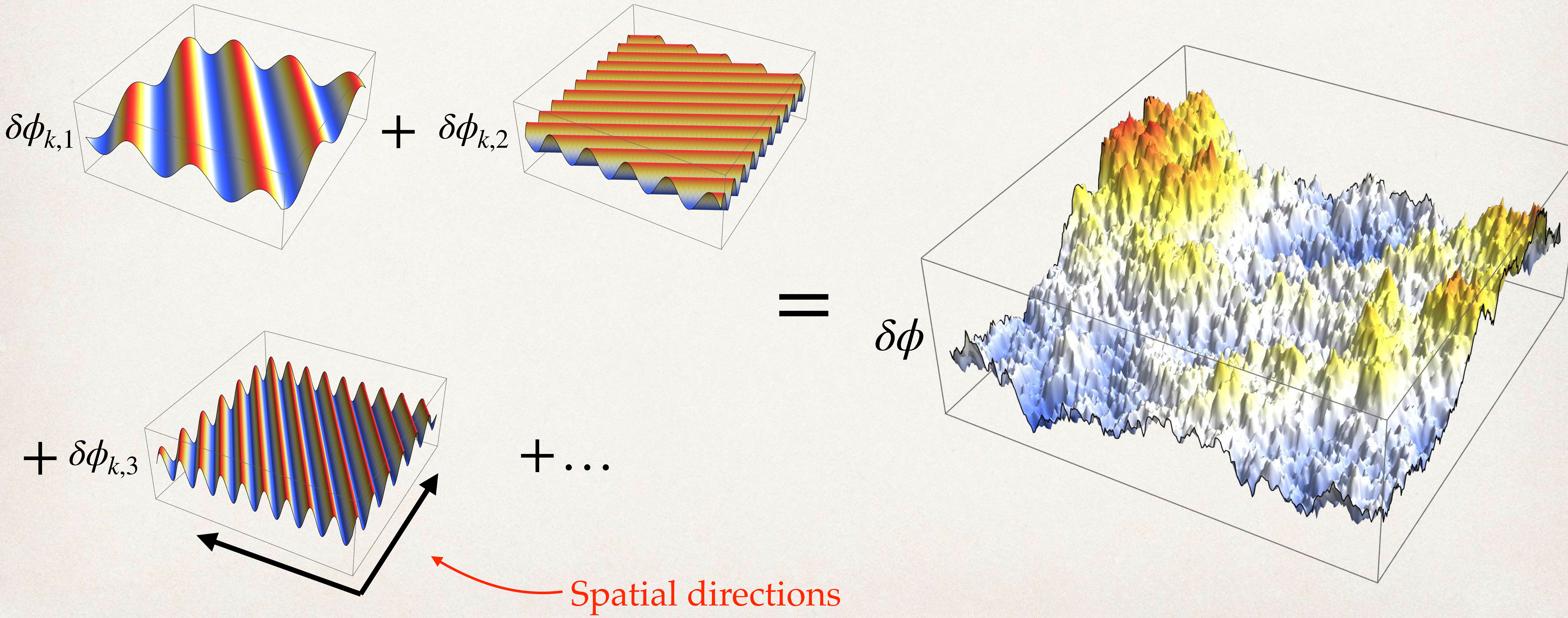



- If we Fourier transform the position variable in the fluctuation to momentum space $k$, then each $k$-mode obeys an equation similar to that of the SHO:

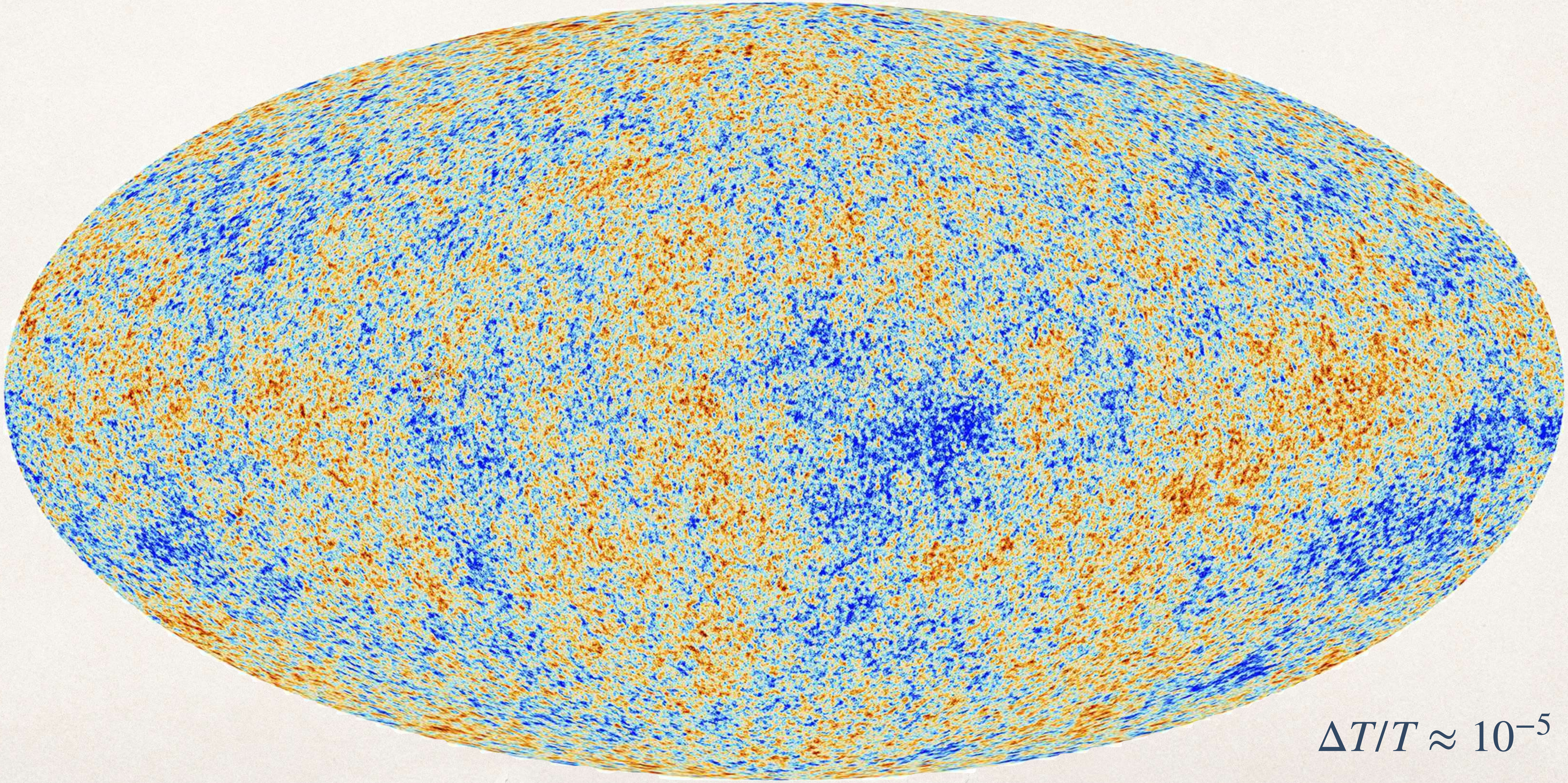
$$\delta\phi_k'' + \omega(t, k)^2 \delta\phi_k = 0$$

- Just like the SHO, quantum mechanics implies that these $k$ -modes $\delta\phi_k(t)$ are typically non-zero when measured!

# Perturbations



# Perturbations



$\Delta T/T \approx 10^{-5}$

# Conclusion

- We discussed a puzzle (called the horizon problem) that stems from observing a uniform CMB despite predictions of the standard Big Bang picture. The latter imply that the CMB is made up of thousands of causally disconnected patches.
- We saw that inflation can remedy this issue by allowing these patches to come into causal contact.
- As a bonus, inflation can also seed fluctuations that become the galaxies, planets and other structures we see around us.
- However, pinning down the microscopic realization of inflation remains an important open problem in theoretical and observational cosmology.

*Thank you for your attention!*